Gigabit DSL

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***Abstract*—** This paper applies multiple input multiple output (MIMO) transmission methods to multi-wire communication systems. Using channel matrices generated from a binder MIMO channel model, a performance assessment of digital subscriber line (DSL) technology based on MIMO transmission methods finds that symmetric data rates of over one gigabit per second are achievable over 4 twisted pairs (category 3) for a 300 meter range. Similar results are also obtained for a "quad" cable. To achieve this data rate, this paper proposes that the source and load be excited using common mode.

***Index Terms*—** MIMO systems, Twisted pair cables, Digital subscriber line, GDSL

I. INTRODUCTION

Gigabit DSL (digital subscriber line) services are technically feasible and tease the imagination of telephone company service providers. Such speeds well in excess of present DSL data rates necessitate a number of topological and signal-processing challenges. Fiber to within 300 meters of the subscriber is presumed, and then the data-carrying limits of 2-4 copper twisted pairs in the remaining final drop segment to the customer using vectored MIMO technology can exceed 1 Gbps symmetric transmission. This paper evaluates such situations and details the basic architecture and structure necessary to implement symmetric gigabit per second DSL (GDSL).

The usual adversary of DSL systems, crosstalk, becomes an enhancement to capacity in the final drop segment where typically 2 to 6 twisted-pairs connect a residence or business with the so-called "service terminal" or "pedestal". While extra copper does not usually exist to connect all customers' 2 to 6 wires back to the central office, extra copper does often exist in the drop segment. Since fiber presumably will connect back to the central office, the need for spare copper capacity in that to-central-office segment is not necessary. Thus, all the wires in the drop segment may be exploited to provide the best achievable data rate to the customer. Coordination of the signals in the final drop with good design practices known as vectoring and bonding allow the entire drop binder of 2 to 6 wires to be viewed as a single transmission path that can have enormous capacity.

Modeling of such a binder is addressed in an earlier paper [3], and Section 2 of this paper investigates the key elements of the 300 meter (or less) drop segment in deriving a reasonable model for the evaluation of its data-carrying capability. Such modeling is known as Multiple Input Multiple Output (MIMO). Section 2 also addresses the potential non-crosstalk noises and assumptions in this paper. Having established a viable model for the drop binder, Section 3 presents well-known vector transmission methods to cancel crosstalk, and even to exploit far end crosstalk (FEXT) [1][2]. Vectored receivers also allow significant reduction of remaining non-crosstalk noises, as is discussed in Section 3. Section 4 then uses specific line parameters to determine that symmetric speeds in excess of 1 Gbps are readily feasible up to at least 300 meters of drop binder on no more than 4 twisted pairs.

II. BINDER MIMO CHANNEL

For a practical multi-twisted pair cable system, the MIMO channel response function can be written as (see [1], Chapter 11 and also the ANSI MIMO model in [5]):

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{1}$$

In (1), $\mathbf{Y}$ is an output column vector whose components are the outputs of the individual transmission lines and $\mathbf{X}$ is an input column vector. The MIMO channel transfer matrix **H** is typically constant (or varies slowing with temperature) if the inputs are synchronized as would be the case in a MIMO GDSL system. $\mathbf{N}$ is non-crosstalk noise that includes thermal noise, radio frequency interference and sometimes impulse noise. (1) represents a vectored DMT (discrete multi-tone) system and DMT has been standardized for most DSL transmission. In addition to clock synchronization, symbol synchronization is achieved through what is known as “digital duplexing” (see [1], Chapter 3). In this case, equation (1) holds independently for each “tone” (frequency sub-channel) of a DMT system. A tone index is not used in (1) to simplify notation, but is presumed. Crosstalk is included in **H**, so for instance for 4 lines, each tone has a **H** matrix that models the crosstalk on that tone between the lines. Because of the common symbol clock, there is insignificant interference and crosstalk between tones of different frequency indices [1].

The binder MIMO channel model of [3] and [5] provide a method to calculate $\mathbf{H}$ from physical system parameters under a variety of excitation and load conditions and for various assumptions of twist lengths, imperfections, and twisted-pair spacing. The model treats a cable as cascades of segments, and each segment can be described by time-invariant transmission-line equations:

$$\begin{aligned} -\frac{d\mathbf{V}}{dz} &= (\mathbf{R} + j\omega\mathbf{L})\cdot\mathbf{I} \\ -\frac{d\mathbf{I}}{dz} &= (\mathbf{G} + j\omega\mathbf{C})\cdot\mathbf{V} \end{aligned} \tag{2}$$

Where **V, I** are the voltage and current vectors, and **R, L, C, G** are the resistance, inductance, capacitance and conductance matrices respectively. Due to twisting of cables and various cable geometric imperfections, the **RLCG** matrices are position dependent and can be computed using basic electromagnetic methods. The transmission line equations can be solved analytically for each segment, and input-output relation for a complete cable can be calculated by considering cascades of segments. To obtain a channel transfer matrix for a complete system, the load and excitation conditions need to be considered. Fig. 1(a) shows the typically encountered differential excitation configuration for a 4 twisted pair cable, where source and load impedances are individually applied to each pair of wires. With differential excitation, a line transfer function can be defined for each pair and FEXT can be defined between any two different pairs. Fig. 1(b) shows a common-mode or "split-pair" excitation configuration, where all 8 wires in 4 pairs are treated equally and one wire is chosen as a common reference [3]. Matrix impedances can be defined which represent the direct and cross couplings between the various voltages and currents. A characteristic impedance matrix $\mathbf{Z}_0$ describes the cable binder, and $\mathbf{Z}_0$ is usually frequency dependent as well as position dependent. Under this configuration, there is some freedom to choose the load impedance matrix $\mathbf{Z}_L$. One practical choice is to select the $\mathbf{Z}_L$ matrix to match $\mathbf{Z}_0$ at the load end ($\mathbf{Z}_L = \mathbf{Z}_0$ (at load end)). Another choice is to select $\mathbf{Z}_L$ as a diagonal matrix. At the source end, $\mathbf{Z}_s$ should be matched to $\mathbf{Z}_0$ of the source end to reduce echo ($\mathbf{Z}_S = \mathbf{Z}_0$ (at source end)). In either the diagonal load impedance or matched load impedance case, the capacity of the binder may be significantly elevated. A numerical example to illustrate such characteristics is presented in Section 4.

Under both differential and common-mode excitations, a MIMO channel matrix can be defined for the channel. If $\mathbf{X}$ is an input voltage vector, and $\mathbf{Y}$ is an output voltage vector, then the channel matrix **H** describes the voltage input-output relationship:

$$\mathbf{V}_L = \mathbf{H} \cdot \mathbf{V}_S \tag{3}$$

where $\mathbf{V}_L = \mathbf{Y}$ is the output voltage vector, and $\mathbf{V}_S = \mathbf{X}$ is the input voltage vector. **H** can be described by a 7x7 matrix for 4 wire-pairs at any given frequency. Assuming differential excitation, a simplified channel matrix, $\mathbf{H}_{sd}$, can be defined by considering the direct transfer function of the wire-pairs and the far end crosstalk between the wire-pairs. For the 4 twisted pairs shown in Fig. 1(a), a *simplified differential* 4x4 channel matrix can be defined as

$$\mathbf{H}_{sd} = \begin{bmatrix} T_1 & Fext_{12} & Fext_{13} & Fext_{14} \\ Fext_{21} & T_2 & Fext_{23} & Fext_{24} \\ Fext_{31} & Fext_{32} & T_3 & Fext_{34} \\ Fext_{41} & Fext_{42} & Fext_{43} & T_4 \end{bmatrix} \tag{4}$$

where the pair-wise direct transfer functions are on diagonal positions of the matrix, and FEXT is in the off-diagonal positions of the matrix. Both the 7x7 and the 4x4 channel matrices can be calculated using the binder MIMO channel model [3]. After the frequency-dependent channel matrix **H** (or $\mathbf{H}_{sd}$) is obtained, digitally duplexed and vector-synchronized discrete multi-tone (DMT) based DSL communication methods (see [1]) can be applied to each tone of the channel.

III. VECTOR TRANSMISSION METHOD AND TRANSCEIVER ARCHITECTURE

This section presents a vector transmission method to estimate the data rate for the MIMO channel described in Section II and the system architecture to achieve the data rate. Methods presented in this section apply to both **H** and $\mathbf{H}_{sd}$, without loss of generality, the symbol **H** is used to represent a channel matrix. For the proposed GDSL system of this paper, coordination of both transmitter and receiver is used, and the data rate can be calculated using the following procedure: **H** is computed as a function of frequency **i.e** $\mathbf{H}(f)$. For any tone *n*, the center frequency f is $f = n \cdot \Delta f$, $n = 1,\ldots,N_{sc}$ where N_{sc} is the maximum number of tones, and Δf is the tone width. The singular values $\lambda_1,\ldots.,\lambda_L$ of $\mathbf{H}(f = n\cdot\Delta f)$ can be found for each tone using singular value decomposition (SVD)[4] as

$$\mathbf{H}(f) = \mathbf{U}(f)\cdot\mathbf{S}(f)\cdot\mathbf{V}^{+}(f) \tag{5}$$

where $\mathbf{S}(f) = diag(\lambda_1,\ldots.,\lambda_L)$, $\lambda_1,\ldots.,\lambda_L$ are diagonal elements of diagonal matrix $\mathbf{S}(f)$, and L is the rank of $\mathbf{H}(f)$. The corresponding "subchannel SNR's" $g_{l,n}$, $l = 1,\ldots,L$, $n = 1,\ldots,N_{sc}$ can be computed:

$$g_{l,n} = g_l(f = n\cdot\Delta f) = \frac{\lambda_{l,n}^2}{\sigma^2} \tag{6}$$

where σ includes all noises but not the crosstalk. If σ is stationary but not "white" or flat across all tones and users, then it has a spatial noise correlation matrix R_{NN} at each tone. A standard noise whitening channel matrix transformation (H to $R_{NN}^{-1/2}H$) may occur at each tone and the effective noise can be assumed to be white.

Execution of the rate adaptive water-filling algorithm [4] over $g_{l,n}$ with known gap Γ subject to a total energy constraint will produce the achievable data rate [4]

$$b = \sum_{l=1}^{L}\sum_{n=1}^{N}\frac{1}{2}\log_2\left(1+\frac{\varepsilon_{l,n}\cdot g_{l,n}}{\Gamma}\right) \tag{7}$$

where $\varepsilon_{total} = \sum_{l=1}^{L}\sum_{n=1}^{N}\varepsilon_{l,n}$ is the total transmit energy available to the system, and $\varepsilon_{l,n}$ is the energy in tone n of the l^{th} component. Practical systems to achieve the target data rate predicted by the above method can be implemented using MIMO based DMT methods. The system architecture for MIMO based DMT systems is similar to a single-line DMT system used in DSL ([1], [4]). Using the SVD in equation (4), the matrix channel in (1) can be reduced to a set of parallel scalar channels [4]:

$$\mathbf{Y} = \mathbf{HX} + \mathbf{N} = \mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^{*}\cdot\mathbf{X} + \mathbf{N}, \tag{8}$$

where $\mathbf{U}$, $\mathbf{V}$ are unitary matrices, $\mathbf{UU}^{*} = \mathbf{I}$, $\mathbf{VV}^{*} = \mathbf{I}$. Multiplying (8) by $\mathbf{U}^{*}$ and defining $\tilde{\mathbf{X}} = \mathbf{V}^{*}\mathbf{X}$

$$\mathbf{U}^{*}\mathbf{Y} = \tilde{\mathbf{Y}} = \mathbf{S}\cdot\tilde{\mathbf{X}} + \tilde{\mathbf{N}}, \text{ and} \tag{9}$$

$$\tilde{y}_{l} = \lambda_{l}\cdot\tilde{x}_{l} + \tilde{n}_{l},\ 1 \le l \le L \tag{10}$$

Equation (10) characterizes a set of parallel scalar channels. These scalar channels are independent of each other, thus the usual DMT based transmission architecture can be directly applied to each scalar channel. Furthermore, the complexity of the MIMO based DMT system would be only moderately increased over L independent DMT systems. This is because the additional complexity only comes from conversion between $\tilde{\mathbf{X}}$ and $\mathbf{X}$, and between $\tilde{\mathbf{Y}}$ and $\mathbf{Y}$ in equation (9). These conversions can be implemented using matrix multiplications. Fig. 2 shows the DMT system architecture for one scalar channel in the frequency domain (matrix multiplications required for the conversions are not shown).

IV. Numerical Examples

The following example illustrates the achievable data rate for 4 twisted pairs over a short length cable. It first uses the methods in Section 2 to obtain the channel matrix, and then uses methods in Section 3 to calculate the achievable data rate. The aggregate data rate of 4 twisted pairs for lengths up to 300-meters is calculated. The cable type is 24AWG and the 4 pairs are assumed to be adjacent to each other.. The twist rates of the 4 pairs are 3.5, 4.6, 4.4, and 4.2 inches. To calculate the symmetric data rate, upstream and downstream data rates are held equal. The total data rate is calculated over all tones and is divided by 2 to get the upstream and downstream data rates. 8192 tones and a tone spacing of 4.3125 kHz are used. Additional parameters used are: 3.8 dB coding gain, 6 dB system margin,

10^{-7} target bit error rate, –140 dBm/Hz background noise and 10 dBm transmission power per line (total power for the system is $10 + 10 \cdot \log_{10}(4)$ = 16 dBm). The aggregate 4-pair data rate is calculated for following source-load configurations: For differential excitation, each pair is differentially terminated with a 100 Ohm impedance. For common-mode MIMO excitation, the sources are chosen to be individually excited ($\mathbf{Z}_S$ is a diagonal matrix); two different load configurations are used: matrix matched impedance ($\mathbf{Z}_L = \mathbf{Z}_0$) or diagonal matrix $\mathbf{Z}_L$. When $\mathbf{Z}_S$ and $\mathbf{Z}_L$ are diagonal matrices, the diagonal elements of these matrices are chosen to be 100 Ohms. The data rate is calculated for both the 7x7 *common mode* channel and the *simplified differential* 4x4 channel. Fig. 3 shows the achievable data rates for these source-load configurations. A few observations are derived from the figure. First, the data rate for a 7x7 matrix channel is higher than that of a 4x4 matrix channel, which is expected from MIMO theory. Second, a common-mode channel has a higher data rate than a differential-mode channel; this can be seen by comparing the data rate of the 7x7 channel with the 4x4 channel. Third, the data rate for a diagonal matrix load is comparable to that of a matrix-matched load. Given the fact that a diagonal matrix load is simpler to implement than a matrix matched load, this result suggests that a diagonal matrix load can be chosen in practical designs. Finally, symmetric (both upstream and downstream) Gigabit/s data rate is achievable by the system, which proves that gigabit DSL (GDSL) is possible for 300 meters with 4 twisted pairs using a frequency band between 0 – 35 MHz.

A closer look at the data rates of the 7x7 channel vs. the 4x4 channel reveals something interesting. The ratio of the data rates is close to 7/4; this is not a coincidence. This ratio is expected according to general MIMO theory [6]. By rough estimation, the data rate for a binder MIMO channel is linearly proportional to the rank of the channel matrix. This observation is useful when other noise situations is considered. For example, in above simulation, –140 dBm/Hz background noise was assumed for both the 7x7 matrix channel and the 4x4 differential channel. For common mode transmission, significantly higher noise can be expected. It is more practical to assume higher noise for 7x7 matrix channel. Using above general MIMO theory, an intuitive estimation of data rate for common mode matrix channel with high noise can be obtained. Normally, the noises over all 7 wires in the common mode will be highly correlated; therefore, it is possible to use the signal from one wire to cancel noise from the other wires. Mathematically, the rank of channel matrix will be 6 instead of 7, and the data rate for such a channel compared to the 4x4 differential channel can be estimated in the neighborhood of 6/4, instead of 7/4. Using the numerical data in Fig. 3, a ratio of 6/4 will give a 1 Gbit/s data rate for the common mode matrix channel at 300 meters.

V. CONCLUSION

This paper shows that by applying MIMO transmission and discrete multi-tone techniques, symmetric data rates in excess of one gigabit per second or GDSL may be possible for 4 twisted pairs over a 300 meter cable length. It also shows that common mode transmission may achieve a higher data rate than differential mode transmission for the same cable binder. The concepts of matrix channel, MIMO transmission as well matrix load impedance matching proposed in this paper may also be applied to gigabit or 10 gigabit Ethernet over other categories of copper twisted-pair channels, and significant performance increases may also be expected in those applications.

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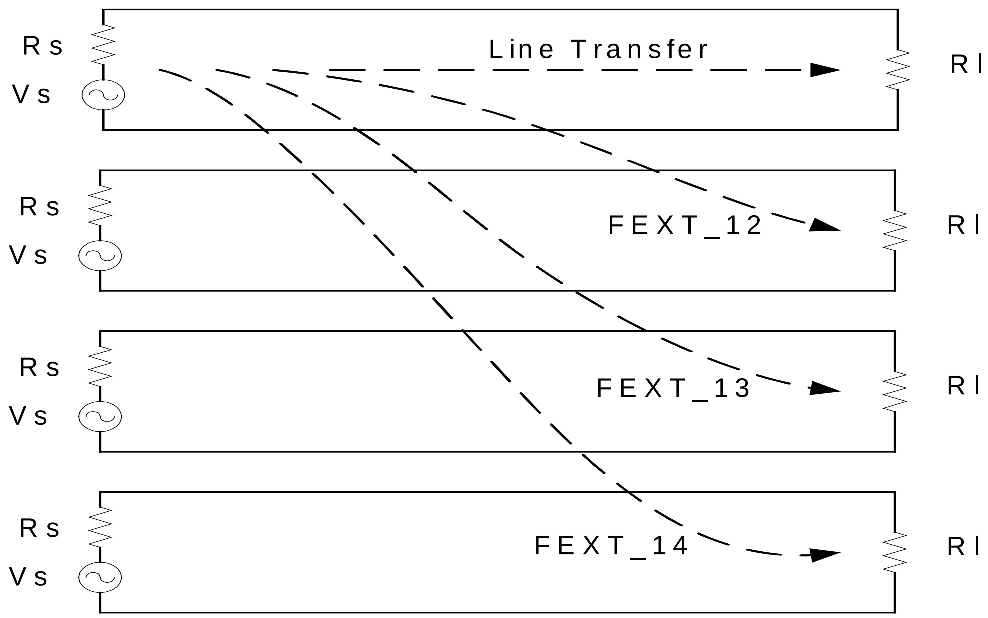

Figure 1(a), Differentially Excited MIMO Channel. Modeled as a 4x4 channel.

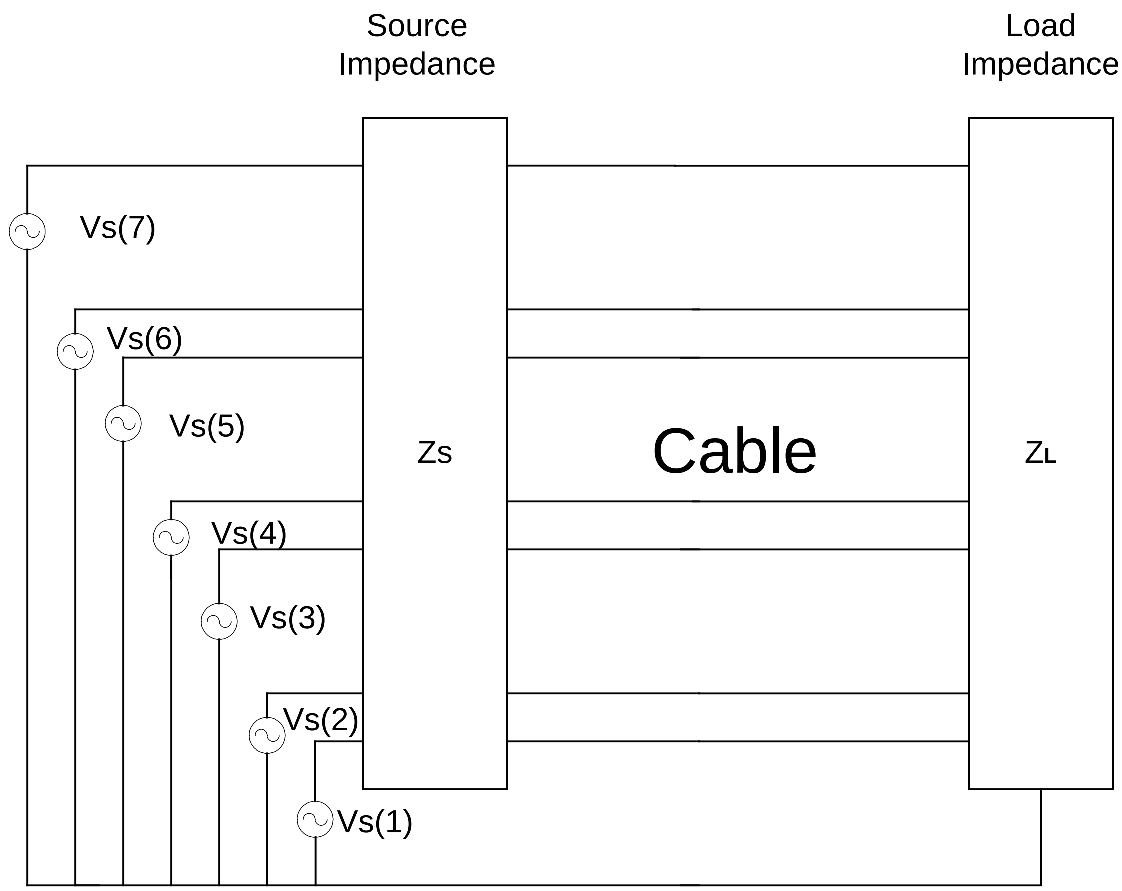

Figure 1(b), Common Mode MIMO Channel. Modeled as a 7x7 channel.

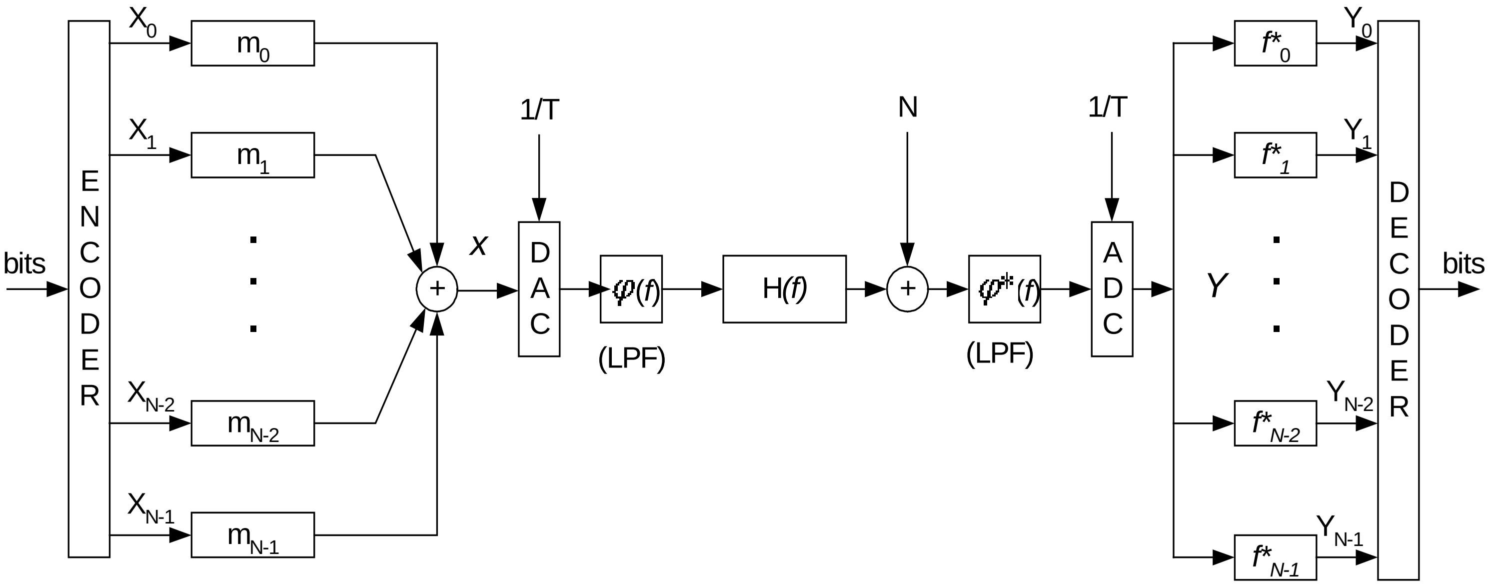

Figure 2, DMT System for one scalar channel, $Y = \tilde{y}_l, H(f) = \lambda_l(f), X = \tilde{x}_l, 1 \le l \le L$. There are N tones, m_i represents the modulation function for tone *i*, and f_i^* is the matched filter for each tone, $0 \le i \le (N-1)$.

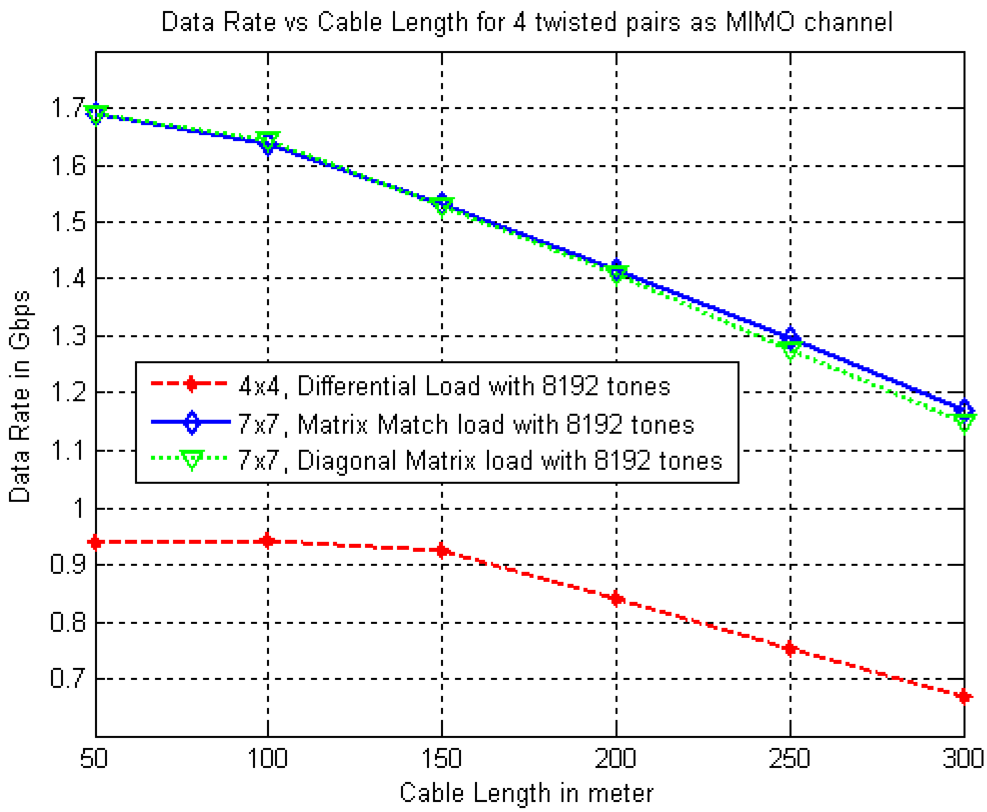

Figure 3, Data Rate vs. Cable Length for 4 Twisted Pairs as a MIMO Channel