

# Tutorial On: Unequal Error Protection in Multicarrier Muti-antenna Systems

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School of Engineering and Science



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# Outline

- 1 Motivations
  - Motivations for UEP, OFDM, and MIMO
- 2 UEP: Bit-Loading
  - Previous Work
  - Proposed Algorithm
- 3 MIMO-OFDM and Eigen Beamforming
  - MIMO Principals
  - Beamforming in MIMO-OFDM
- 4 Simulation Results
  - Simulation Parameters
  - UEP Adaptive MIMO-OFDM Results
- 5 Conclusions

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# Realizing UEP

- ◇ **UEP**: invokes the need for **non-uniform** error protection.
- ◇ **OFDM**: suitable for **adapting individual subcarriers** using different data rates, code rates, and powers
- ◇ **MIMO**: has high **multiplexing** gain and allows for channel layering.
- ◇ **UEP MIMO-OFDM**: devotes an arbitrary number of bits to different classes, eigenbeams, and subcarriers

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# Why UEP ?



- ◇ Source encoders of some applications **deliver data of different importance**.
- ◇ Matching the **channel variations** to enhance **performance** and **spectral efficiency**.
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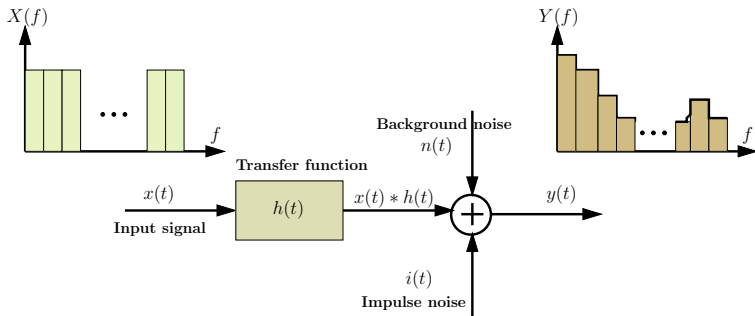
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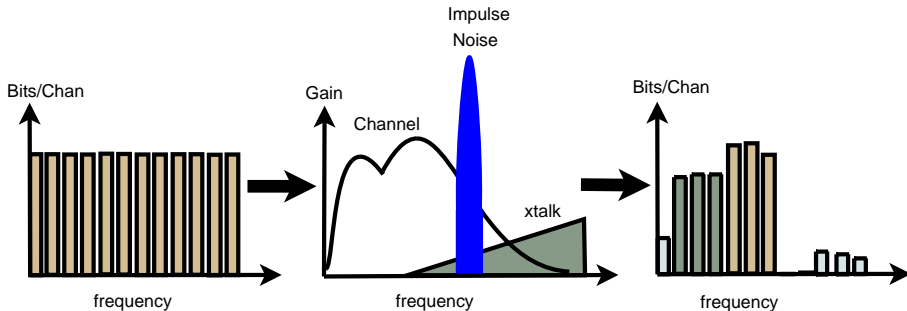
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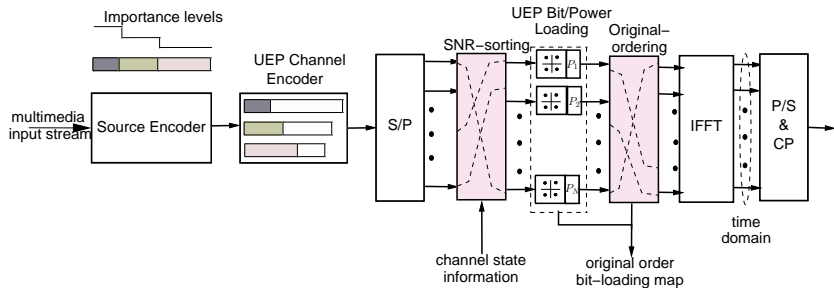


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# UEP Schemes in MCM

Adapt coding scheme/rate (i.e., use puncturing or pruning)

Adapt bit/power loading and Physical Transport (e.g. MIMO Channel)



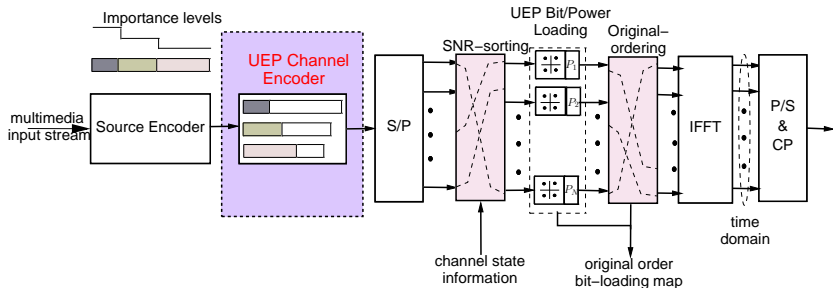
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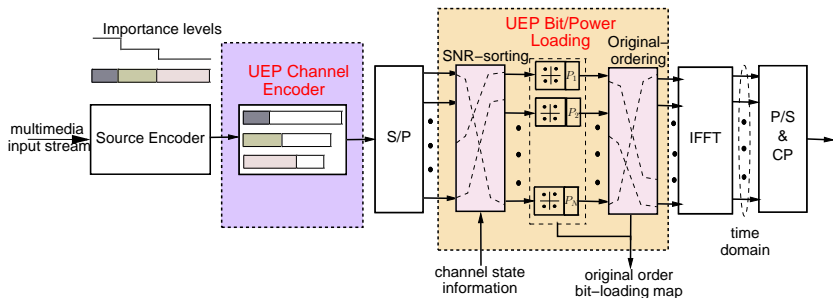
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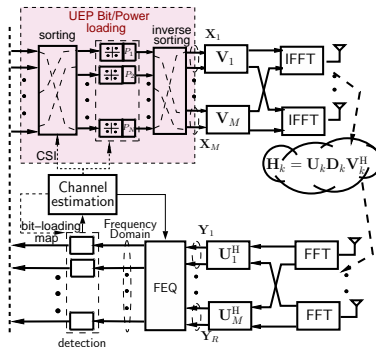
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# Principals

## Modified Shannon's Capacity:

$$b_k = \log_2 \left( 1 + \frac{\text{SNR}_k}{\gamma} \right)$$

### Three conceptual problems:

- Bit-rate maximization problem (BRMP)
- Power minimization problem (PMP)
- Probability of error minimization problem (PEMP)

$$\max_{\hat{b} \in Z} \sum_{k=0}^{N-1} \hat{b}_k$$

subject to

$$\sum_{k=0}^{N-1} P_k(\hat{b}_k) < P_T$$

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# Bit-Loading Algorithms

## Bit-loading solutions:

- **Optimum**: add bits to the locations of minimum incremental power, e.g.: **Hughes-Hartogs** and **Campello**
- **Sub-optimum**: based on Shannon capacity (**Chow et al.**) or probability of error minimization (**Fischer-Huber and Yu-Willson**)

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### MIMO UEP Bit-Loading (BRMP):

$$b_{k,l}^{(j)} = \log_2 \left( 1 + \frac{P_{k,l}^{(j)} \cdot \lambda_{k,l}^{(j)}}{\sigma^2 \cdot \gamma^{(j)}} \right)$$

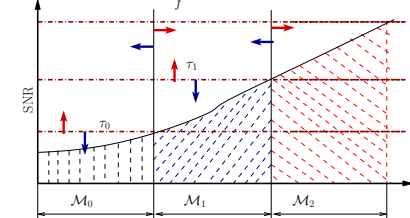
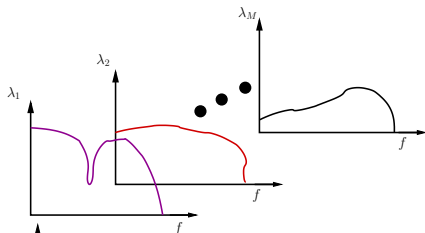
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# UEP Bit-Loading and SNR-Sorting Algorithms

- Compute  $b_{k,l}^{(j)}$  using  $\gamma^{(j)} = \gamma_0 - j \cdot \Delta\gamma$ , then adjust  $\mathcal{M}^{(j)}$  iteratively until  $\sum_{k,l} b_{k,l}^{(j)} = T^{(j)}$  or maximum iteration
- If  $B_T$  is not achieved, update  $\gamma_0$  and recompute. If maximum iterations, add/subtract bits according to  $\Delta b_k^{(j)}$
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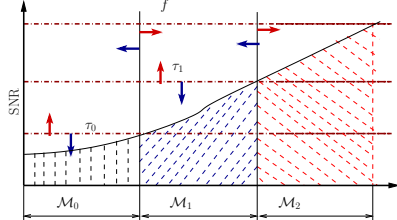
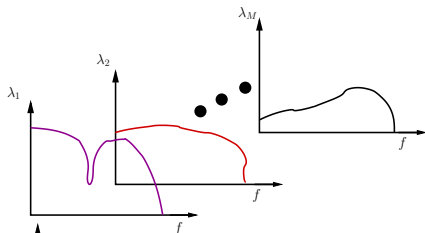
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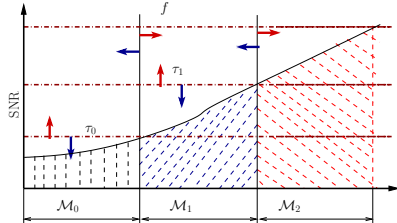
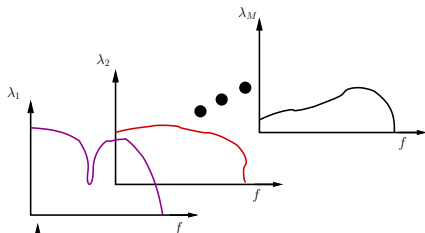


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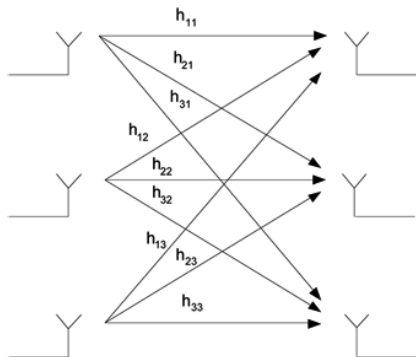
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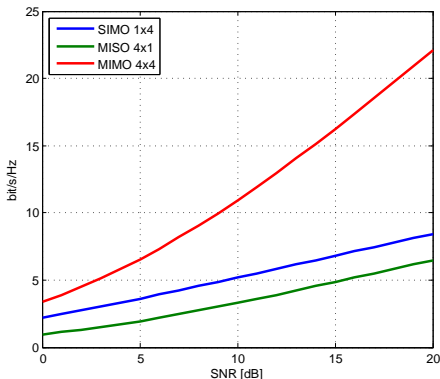
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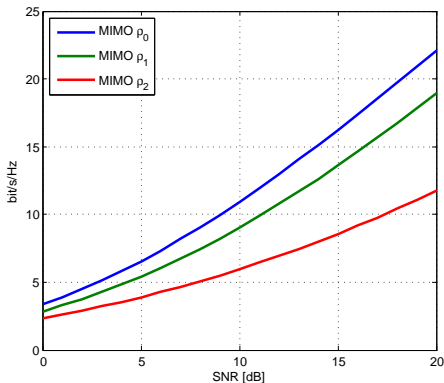


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bits/sec/Hz

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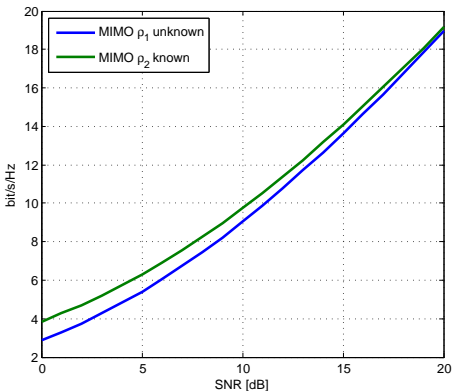
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where  $\mathbf{Q} = E[x * x^H]$

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$$C_E = \sum_{i=1}^M \log_2 \left( 1 + \frac{\rho_{WF}}{N_T} \lambda_i \right)$$

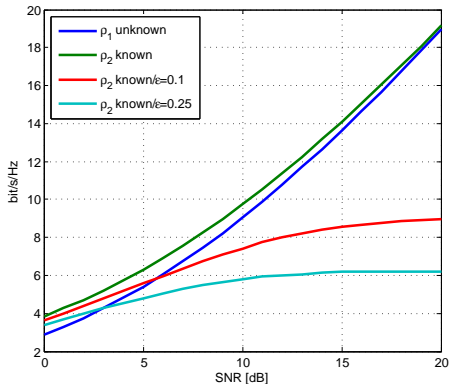
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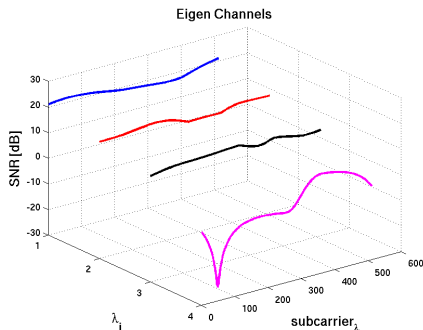
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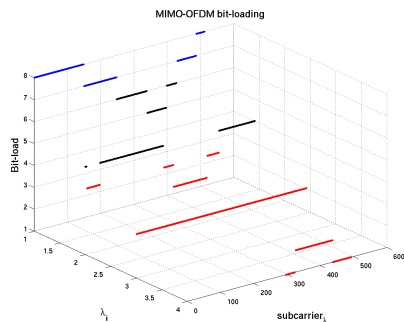


# Eigen Channels Representation

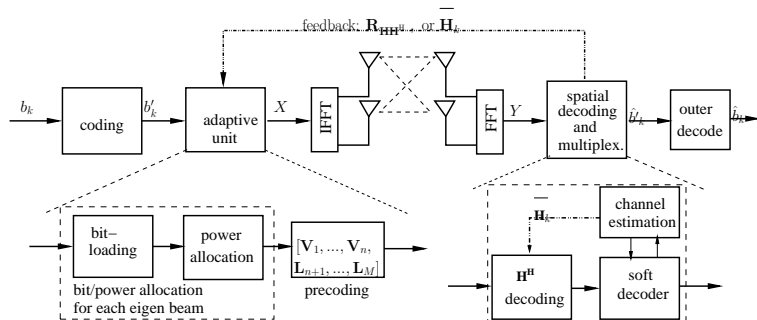
## Eigen channels (modes)



## Bit-loading for eigen channels



# Channel side information feedback



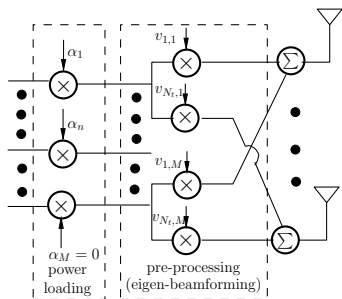
## CSI feedback:

- Channel mean:  $\hat{\mathbf{H}} = \mathbf{H} - \varepsilon_e$ ,  
 $\overline{\mathbf{U}} \mathbf{D}^{\frac{1}{2}} \overline{\mathbf{V}}^{\mathbf{H}}$
- Channel correlation:  
 $R_{\hat{\mathbf{H}}\hat{\mathbf{H}}} = E\{\hat{\mathbf{H}}\hat{\mathbf{H}}^*\} = \overline{\mathbf{V}} \mathbf{D} \overline{\mathbf{V}}^{\mathbf{H}}$

## CSI uncertainty:

- Channel estimation error
- Quantization error
- errors included by the feedback channel
- Variation during channel feedback

# Beamforming Scheme



The rank =  $M$  &  $0 < n \leq M - 1$

$$\therefore \bar{\mathbf{V}} = [\bar{\mathbf{V}}_1 \bar{\mathbf{V}}_2],$$

where  $\bar{\mathbf{V}}_1 = [v_1, \dots, v_n]$  and

$$\bar{\mathbf{V}}_2 = [0_{n+1}, \dots, 0_M].$$

## Eigen beamforming selection

- full-beamforming (full-BF) at  $n = M$
- suppress weaker eigenbeams
- shorter BF length due to antenna correlation or CSI errors
  - Direct BF:  $\bar{\mathbf{V}}_1$  are adjacent columns.
  - Selecte BF:  $\bar{\mathbf{V}}_1$  are selected to minimize interference

# Beamforming Analysis

CSI error:  $\hat{\mathbf{H}}_k = \bar{\mathbf{H}}_k + \Xi_k$   
 where  $\Xi_k \sim \mathcal{CN}(0, \sigma_{\Xi}^2)$   
 the received vector:

$$\begin{aligned} \mathbf{Y}_k &= \hat{\mathbf{H}}_k \bar{\mathbf{V}}_k \mathbf{P}^{1/2} \mathbf{X}_k + n_k \\ &= \underbrace{\hat{\mathbf{U}}_k \hat{\mathbf{D}}_k \hat{\mathbf{V}}_k^* \bar{\mathbf{V}}_k \mathbf{P}^{1/2}}_{\mathbf{T}_k} \mathbf{X}_k + \eta_k, \end{aligned}$$

ZF-MRC detection:

$$\mathbf{W} = \{\mathbf{T}^* \mathbf{T}\}^{-1} \mathbf{T}^H$$

$$\hat{\mathbf{x}} = \mathbf{W} \mathbf{y}$$

MMSE-MRC detection:

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# Channel Model and System Parameters:

- ◇ **Channel:** MIMO Rayleigh fading channel with different correlation models
- ◇ **MIMO Parameters:**  $4 \times 4$  MIMO-OFDM system with 512 subcarriers for each beam
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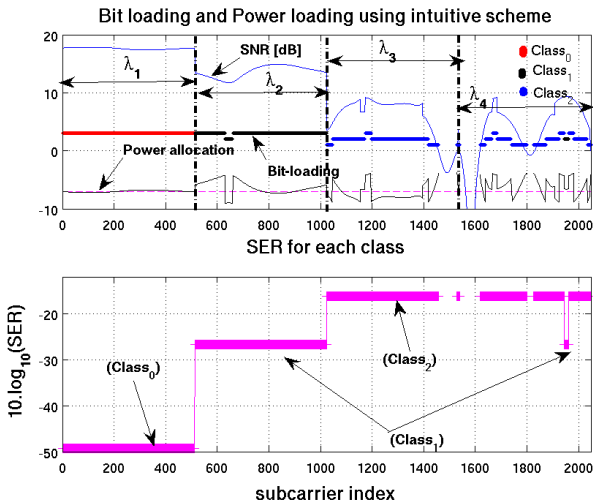
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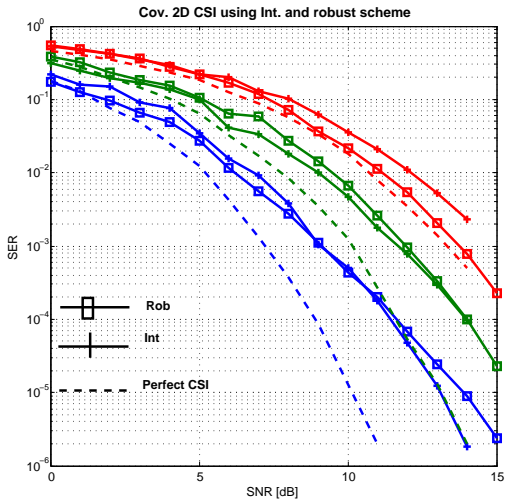
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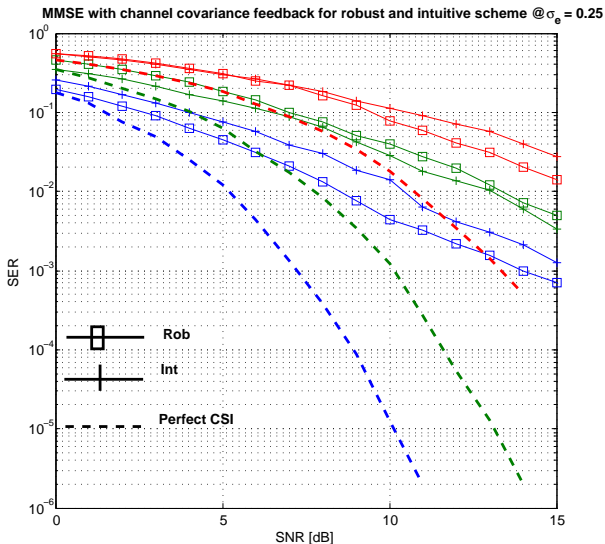
# UEP Bit Power Allocation for perfect CSI:



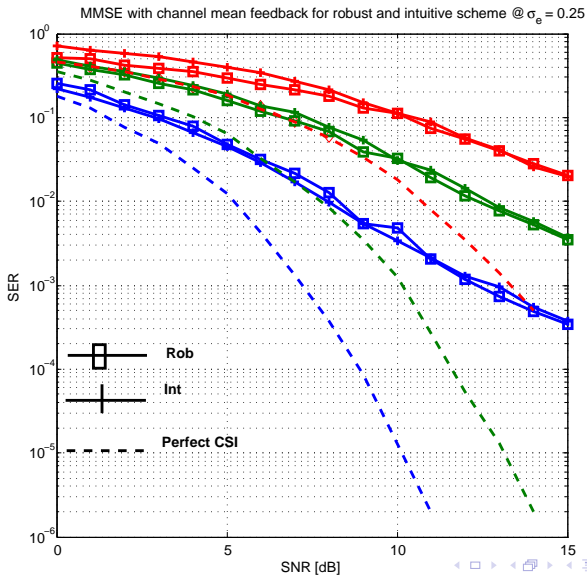
# perfect and imperfect CSI (2D results @ $\varepsilon_e = 0.1$ )



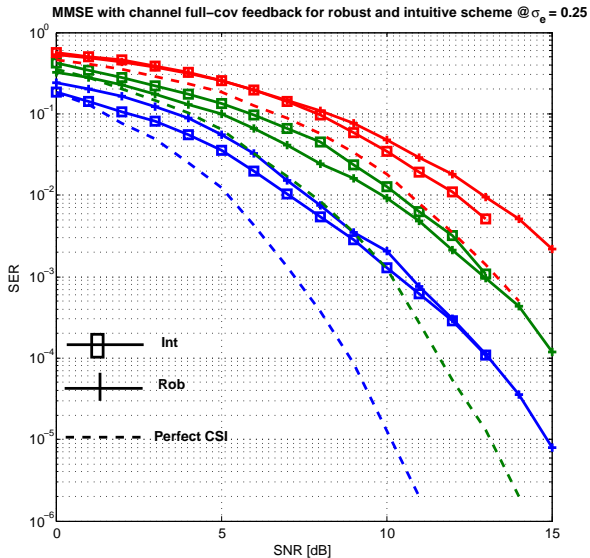
# Different CSI errors (2D results @ $\varepsilon_e = 0.25$ ):



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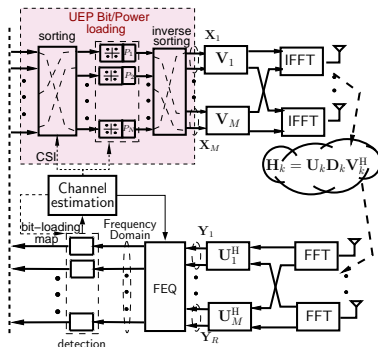


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- We described an UEP bit-allocation scheme for MIMO-OFDM
- Exploit channel layering using SVD, thereby realize UEP
- Allows for arbitrary margins, error probabilities, and bit-rates
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## Ongoing Research:

We are studying the combination of spatial equalizers, IC, beamforming, and STBC to minimize the CSI errors effect.

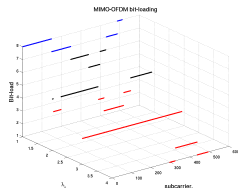
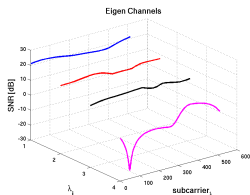


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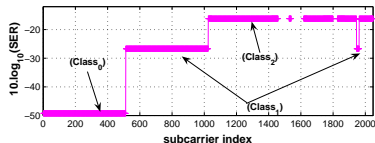
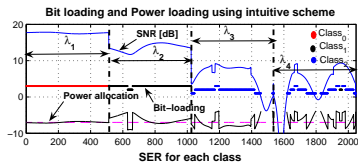
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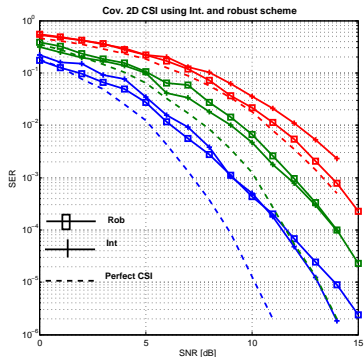
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## Questions!

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# CSI Error Effect

