Tutorial On: Unequal Error Protection in Multicarrier Multi-antenna Systems

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School of Engineering and Science

Summer Academy, 2007
1 Motivations
   - Motivations for UEP, OFDM, and MIMO

2 UEP: Bit-Loading
   - Previous Work
   - Proposed Algorithm

3 MIMO-OFDM and Eigen Beamforming
   - MIMO Principals
   - Beamforming in MIMO-OFDM

4 Simulation Results
   - Simulation Parameters
   - UEP Adaptive MIMO-OFDM Results

5 Conclusions
1. **Motivations**
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2. **UEP: Bit-Loading**
   - Previous Work
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3. **MIMO-OFDM and Eigen Beamforming**
   - MIMO Principals
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4. **Simulation Results**
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   - UEP Adaptive MIMO-OFDM Results

5. **Conclusions**
Realizing UEP

- **UEP**: invokes the need for non-uniform error protection.

- **OFDM**: suitable for adapting individual subcarriers using different data rates, code rates, and powers.

- **MIMO**: has high multiplexing gain and allows for channel layering.

- **UEP MIMO-OFDM**: devotes an arbitrary number of bits to different classes, eigenbeams, and subcarriers.
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Why UEP?

- Source encoders of some applications deliver data of different importance.
- Matching the channel variations to enhance performance and spectral efficiency.
- The different error sensitivities of different communication devices, e.g., PDAs, laptops, etc.
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Why Multicarrier?

The available bandwidth is divided into $N$ individual sub-channels.

Due to its suitability for adapting individual subcarriers with different data rates, code rates, and power according to channel conditions.
Motivations for UEP, OFDM, and MIMO

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Motivations for UEP, OFDM, and MIMO

UEP Schemes in MCM

[Diagram]

UEP coding Layer
- Adapt coding scheme/rate (i.e., use puncturing or pruning)

UEP Physical Layer
- Adapt bit/power loading and Physical Transport, e.g.: MIMO Channel

Source Encoder
- Multimedia input stream

UEP Channel Encoder
- Importance levels

SNR-sorting
- Order

Original-ordering
- Original order

Bit/Power Loading
- Channel state information

UEP Bit/Power Loading
- Original bit-loading map

S/P & CP
- P/S

IFFT
- Time domain

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\[ H_k = U_k D_k V_k^H \]

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Unequal Error Protection in MC-MIMO

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Modified Shannon’s Capacity:

\[ b_k = \log_2 \left( 1 + \frac{\text{SNR}_k}{\gamma} \right) \]

Three conceptual problems:
- Bit-rate maximization problem (BRMP)
- Power minimization problem (PMP)
- Probability of error minimization problem (PEMP)

Maximize:

\[ \max_{\hat{b} \in Z} \sum_{k=0}^{N-1} \hat{b}_k \]

Subject to:

\[ \sum_{k=0}^{N-1} P_k(\hat{b}_k) < P_T \]
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\[
\begin{align*}
\min_{\hat{b} \in \mathbb{Z}} & \sum_{k=0}^{N-1} P_k \\
\text{subject to} & \\
\sum_{k=0}^{N-1} \hat{b}_k &= B_T \\n\sum_{k=0}^{N-1} P_k &\leq P_T
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Minimize

\[ \min_{\hat{b} \in \mathbb{Z}} \sum_{k=0}^{N-1} P_{e,k} \]

Subject to

\[ \sum_{k=0}^{N-1} \hat{b}_k = B_T \quad \text{and} \quad \sum_{k=0}^{N-1} P_k \leq P_T \]
Bit-Loading Algorithms

Bit-loading solutions:
- **Optimum**: add bits to the locations of minimum incremental power, e.g.: Hughes-Hartogs and Campello
- **Sub-optimum**: based on Shannon capacity (Chow et al.) or probability of error minimization (Fischer-Huber and Yu-Willson)

**Bit-Loading by Chow (BRMP):**

$$b_k = \log_2 \left( 1 + \frac{\text{SNR}_k}{\gamma} \right)$$

**Quantization Error:**

$$\hat{b}_k = \left\lfloor b_k + 0.5 \right\rfloor_{0}^{b_{\text{max}}}$$

$$\Delta b_k = b_k - \hat{b}_k$$
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**MIMO UEP Bit-Loading (BRMP):**

\[
b_{k,l}^{(j)} = \log_2 \left( 1 + \frac{P_{k,l}^{(j)} \cdot \lambda_{k,l}^{(j)}}{\sigma^2 \cdot \gamma^{(j)}} \right)
\]

**Quantization Error:**

\[
\hat{b}_{k,l}^{(j)} = \left\lfloor b_{k,l}^{(j)} + 0.5 \right\rfloor_{0}^{b_{\text{max}}}
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\[
\Delta b_{k,l}^{(j)} = b_{k,l}^{(j)} - \hat{b}_{k,l}^{(j)}
\]
Proposed Algorithm

Compute $b_{k,l}^{(j)}$ using $\gamma^{(j)} = \gamma_0 - j \cdot \Delta \gamma$, then adjust $M^{(j)}$ iteratively unit $\sum_{k,l} b_{k,l}^{(j)} = T^{(j)}$ or maximum iteration.

If $B_T$ is not achieved, update $\gamma_0$ and recompute. If maximum iterations, add/subtract bits according to $\Delta b_{k}^{(j)}$.

The power is allocated according to SER. If the target SER is not fulfilled, reduce the total rate.
Compute $b^{(j)}_{k,l}$ using $\gamma^{(j)} = \gamma_0 - j \cdot \Delta \gamma$, then adjust $\mathcal{M}^{(j)}$ iteratively until
$$\sum_{k,l} b^{(j)}_{k,l} = T^{(j)}$$ or maximum iteration

If $B_T$ is not achieved, update $\gamma_0$ and recompute. If maximum iterations, add/subtract bits according to $\Delta b^{(j)}_{k}$

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**UEP Bit-Loading and SNR-Sorting Algorithms**

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Potential of MIMO wireless links:

- substantial improvement in QoS/throughput
- Effectively exploit multipath
- Scalability and adaptation
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\[ C_E = E \left[ \det \left( \log_2 \left\{ I_{M_R} + \frac{\rho}{N_T} HH^H \right\} \right) \right] \text{ bits/sec/Hz} \]
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Where \( Q = E[\mathbf{x} \circ \mathbf{x}^H] \)
Potential of MIMO wireless links:

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\[ C_E = \sum_{i=1}^{M} \log_2 \left( 1 + \frac{\rho_{WF}}{N_T} \lambda_i \right) \text{bits/sec/Hz} \]

where \( \text{tr}(Q) \leq \rho_{WF} \)
\[ \rho_{WF} = \sum_{i=1}^{M} \left( \mu - \lambda_i^{-1} \right)^+ \]
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Eigen Channels Representation

Eigen channels (modes)  Bit-loading for eigen channels

Eigen Channels

Bit-loading for eigen channels

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CSI feedback:

- Channel mean: $\hat{H} = H - \varepsilon_e$, $UD^{1/2} V^H$
- Channel corelation: $R_{\hat{H}H}\hat{H} = E\{\hat{H}\hat{H}^*\} = VDV^H$

CSI uncertainty:

- Channel estimation error
- Quantization error
- Errors included by the feedback channel
- Variation during channel feedback
Beamforming Scheme

Eigen beamforming selection

- full-beamforming (full-BF) at $n = M$
- suppress weaker eigenbeams
- shorter BF length due to antenna correlation or CSI errors
  - Direct BF: $\bar{V}_1$ are adjacent columns.
  - Selecte BF: $\bar{V}_1$ are selected to minimize interference

The rank $= M$ & $0 < n \leq M - 1$

$\vdots \bar{V} = [\bar{V}_1 \bar{V}_2]$, 
where $\bar{V}_1 = [v_1, \ldots, v_n]$ and 
$\bar{V}_2 = [0_{n+1}, \ldots, 0_M]$. 
CSI error: $\hat{H}_k = H_k + \Xi_k$
where $\Xi_k \sim \mathcal{CN}(0, \sigma^2_{\Xi})$
the received vector:

$$Y_k = \hat{H}_k V_k P^{1/2} X_k + n_k$$

$$= \hat{U}_k \hat{D}_k \hat{V}^*_k \bar{V} P^{1/2} X_k + \eta_k ,$$

ZF-MRC detection:

$$W = \{T^* T\}^{-1} T^H$$

$$\hat{x} = Wy$$

MMSE-MRC detection:

$$W = \{T^* T + \sigma^2_N I\}^{-1} T^H$$

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**Channel Model and System Parameters:**

- **Channel:** MIMO Rayleigh fading channel with different correlation models
- **MIMO Parameters:** $4 \times 4$ MIMO-OFDM system with 512 subcarriers for each beam
- **Bit-loading:** the maximum allowed bits per subchannel is 8
- **UEP Application:** 3 classes, $\Delta \gamma^{(j)} = 3 \text{ dB}$, $T^{(j)} = 1024 \text{ bits}$
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Simulation Results

UEP Bit Power Allocation for perfect CSI:

Bit loading and Power loading using intuitive scheme

SER for each class

10 \log_{10} (SER)

subcarrier index
perfect and imperfect CSI (2D results @ $\varepsilon_e = 0.1$)
Different CSI errors (2D results \( @\varepsilon_e = 0.25 \)):
Simulation Results

UEP Adaptive MIMO-OFDM Results

Different Beamforming Techniques (full beamforming):

MMSE with channel mean feedback for robust and intuitive scheme \( \sigma_e = 0.25 \)

- Rob
- Int
- Perfect CSI

SNR [dB]
SER

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- Exploit channel layering using SVD, thereby realize UEP
- Allows for arbitrary margins, error probabilities, and bit-rates
- Selected beamforming is a practical solution for suppressing CSI errors.

Ongoing Research:

We are studying the combination of spatial equalizers, IC, beamforming, and STBC to minimize the CSI errors effect.
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Questions!
CSI Error Effect

Hist. of full beamforming NO CSI ERRORS

Hist. of full beamforming with CSI errors of $\sigma^2 = 0.1$

Hist. of selected beamforming (1,2) with CSI errors of $\sigma^2 = 0.1$

Hist. of selected beamforming (1,3) with CSI errors of $\sigma^2 = 0.1$