Multicarrier Modulation

Summer Academy at JUB

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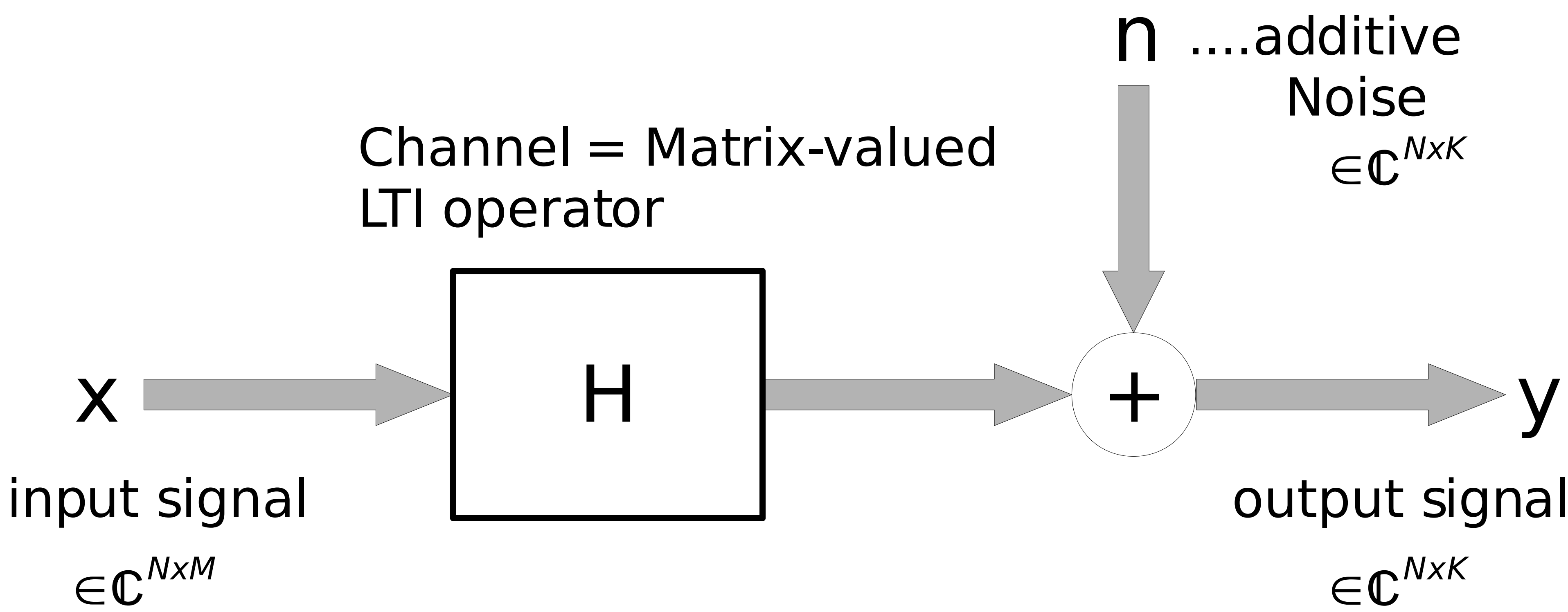
Overview

- Digital Communication: Problem Setup
- Algebraic constraints for signal design:
  - Orthonormal, Biorthogonal, Overcomplete Systems
- Structural constraints for signal design:
  - Useful tilings of the time-frequency plane
- Latency Splitup: Coding versus Modulation
- Practical Examples: VDSL2, WIMAX
- Open Issues:
  - Semiblind channel estimation
  - Autonomous spectrum management

Problem Setup

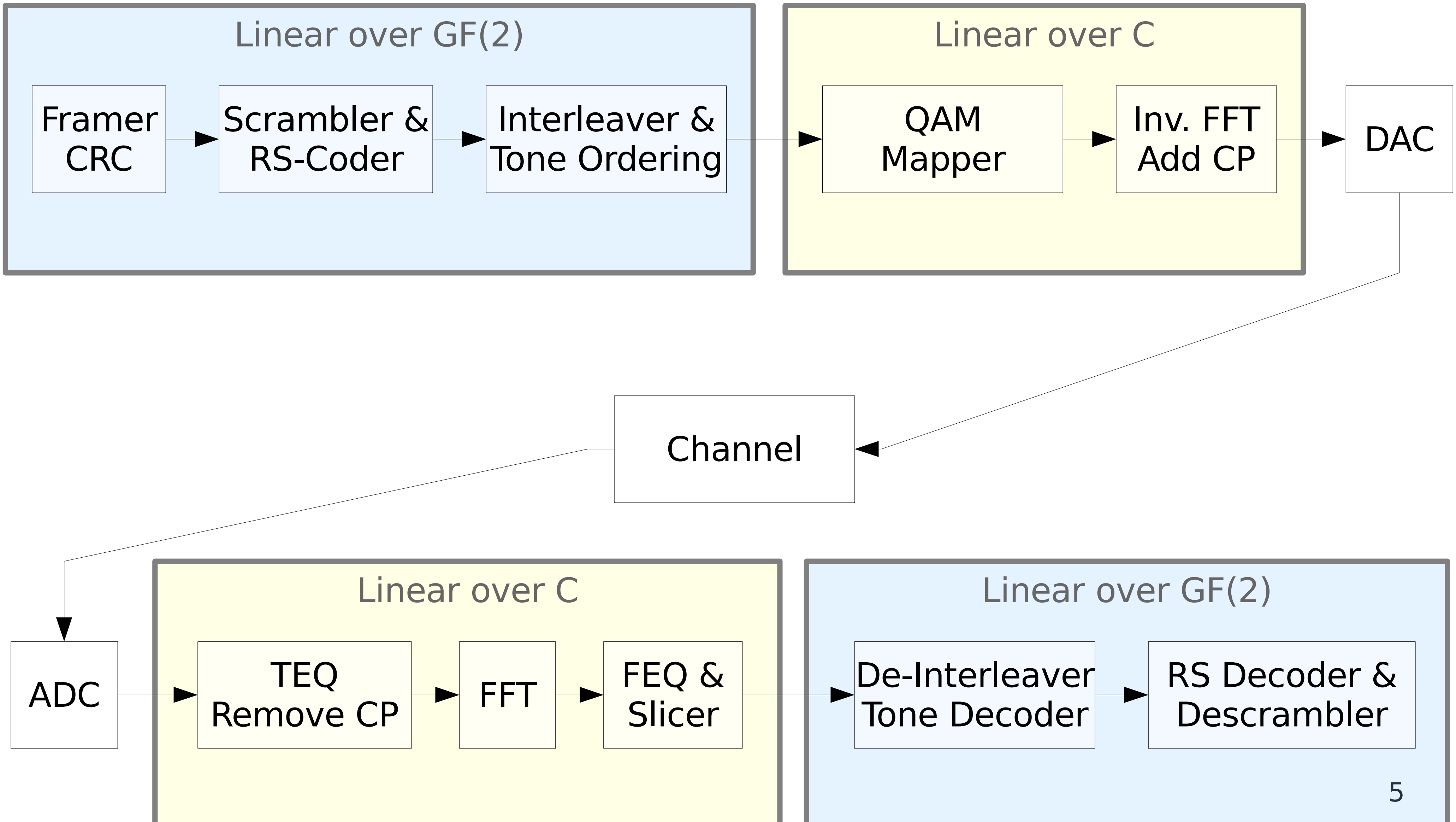


$$y = H x + n$$

Pioneering MCM Literature

- Peled, Ruiz, Frequency domain data transmission using reduced computational complexity algorithms, 1980.
- Ruiz, Cioffi, Casturia, DMT with Coset Coding for the Spectrally Shaped Channel, 1987.
- Bingham, Multicarrier Modulation for Data Transmission: An Idea Whose Time has Come, 1990.
- Wozencraft, Moose, Modulation and Coding for In-Band DAB using Multi-Frequency Modulation, 1991.
- Chow, Tu, Cioffi, A DMT transceiver System for HDSL Applications 1991.
- LeFloch, Alard, Berrou, Coded orthogonal frequency division multiplex 1995.

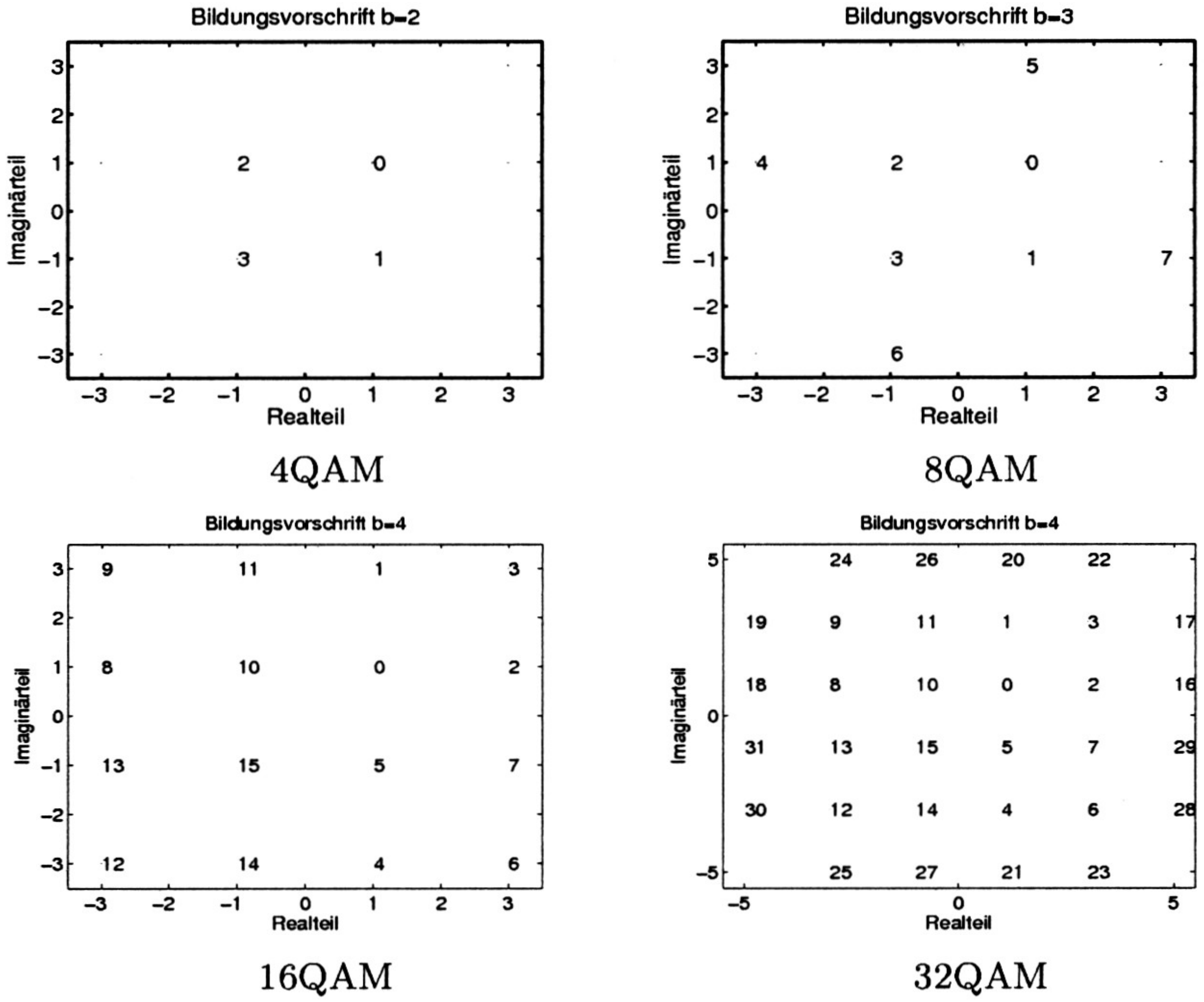
MC-Modem Building Blocks



Why Complex Numbers ?

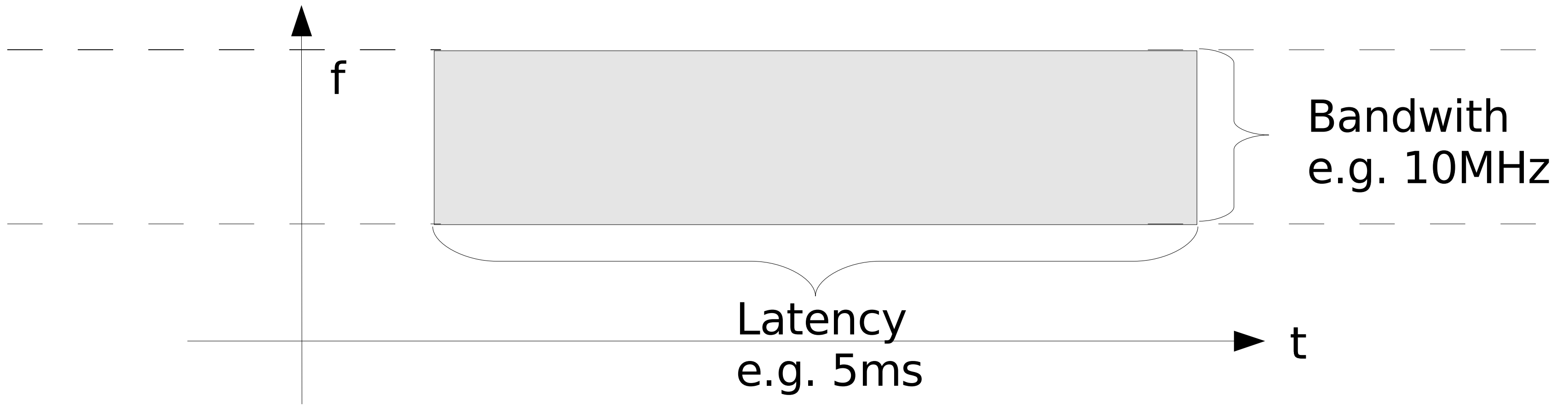
- Real-valued convolution operators are diagonalized by the complex valued Fourier transform
- Modern linear algebra allows to handle circulant matrices directly over the reals: Singular value decomposition (SVD)
- One needs two copies of the time-frequency plane: the sine-copy and the cosine-copy => slightly inconvenient
- This is how engineers handle the transmission of complex signals; their terminology is:
  - cosine-copy = „in-phase component“
  - sine-copy = „quadrature component“

Typical QAM Mappings



Signal Design Problem

- *Finite* amount of *discrete* data mapped onto signal within predefined TF-subspace (a large one typically):



- Receiver needs to recover this information although the signal is subject to channel distortion and additive noise
- Divide-and-conquer: Split up the signal space into rank-one subspaces => series expansion of transmit signal

$$x(n) = \sum_k c_k g_k(n)$$

Orthonormal bases

- The standard setup for digital communication and information theory
- Transmission signal: $x(n) = \sum_k c_k g_k(n)$
- Receiver Detection: $c_k = \langle x, g_k \rangle$
- ONB condition: $\langle g_k, g_l \rangle = \delta_{k,l} \quad \forall k, l$

Biorthogonal bases

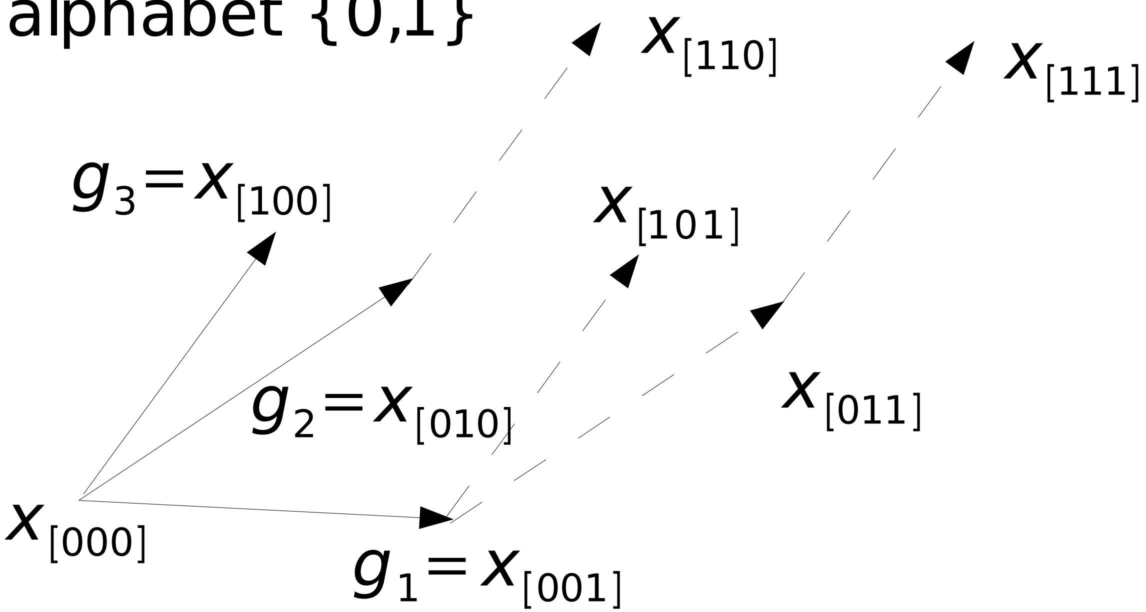
- In a Hilbert space setting unique recovery of coefficients in a series expansion requires Riesz bases rather than ONBs
- The coefficents are recovered with the help of a biorthogonal basis
- Transmission signal: $x(n)=\sum_k c_k g_k(n)$
- Receiver Detection: $c_k=\langle x, f_k\rangle$
- Biorthogonality Condition: $\langle g_k, f_l\rangle=\delta_{k,l} \quad \forall k, l$
- (Beware: Communication Engineers use the phrase biorthogonality in a totally disjoint meaning)

Overcomplete Systems

- Consider a finite, discrete setting e.g. $\mathbb{C}^N$
- M vectors of length N < M
- These vectors are necessarily linear dependent
  => coefficents undetermined in a Hilbert space setting
- However, for finite alphabet coefficients recovery is in general possible
- Example: 3 Random Vectors on R^2 and the binary coefficents with alphabet {0,1}

Overcomplete Systems (ctd.)

- Example: 3 Random Vectors on R^2 and the binary coefficients with alphabet {0,1}

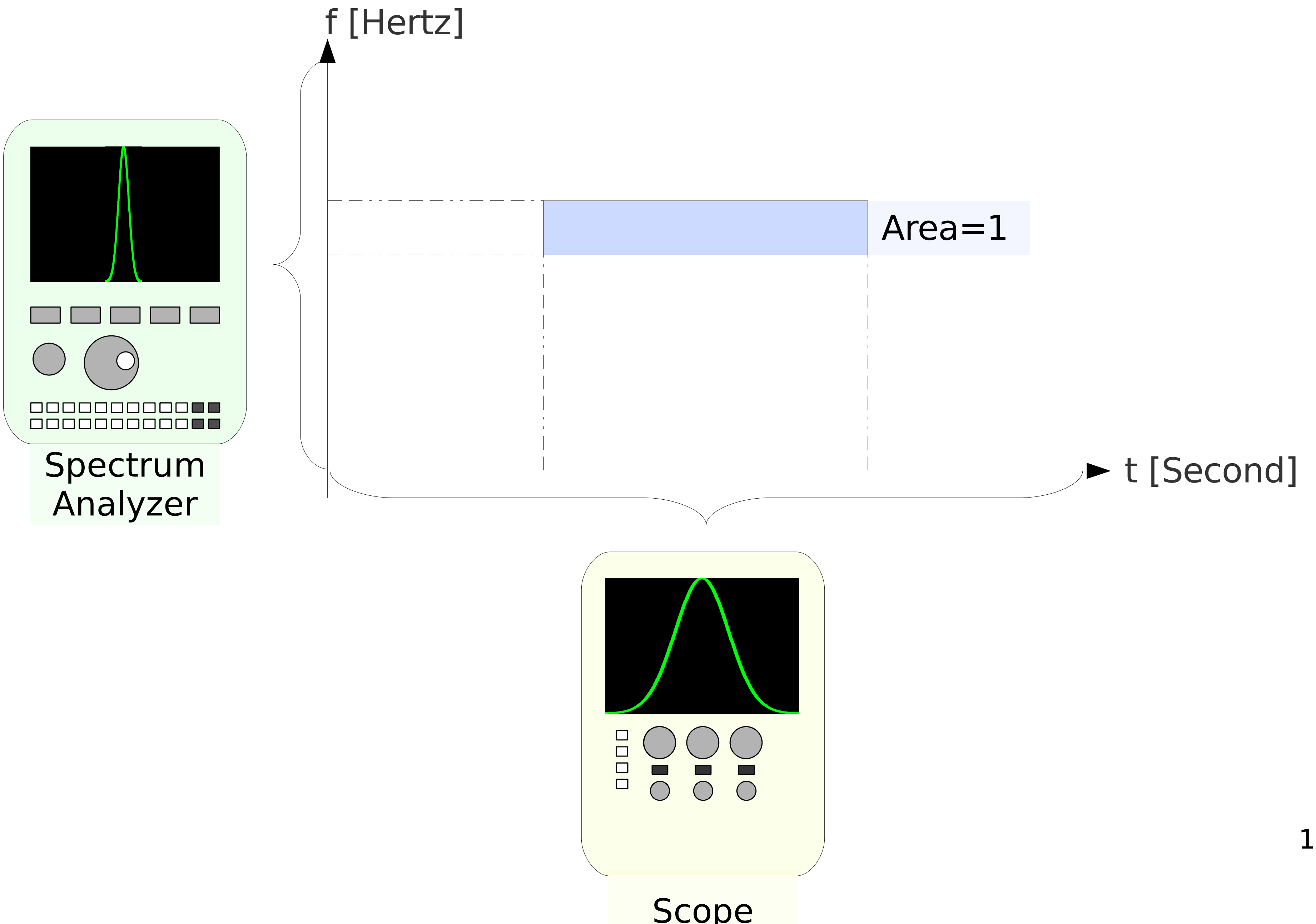


- Remark 1: Symmetrical choices lead to collisions
- Remark 2: Noise-free observation of at least one real number is *pure* mathematics

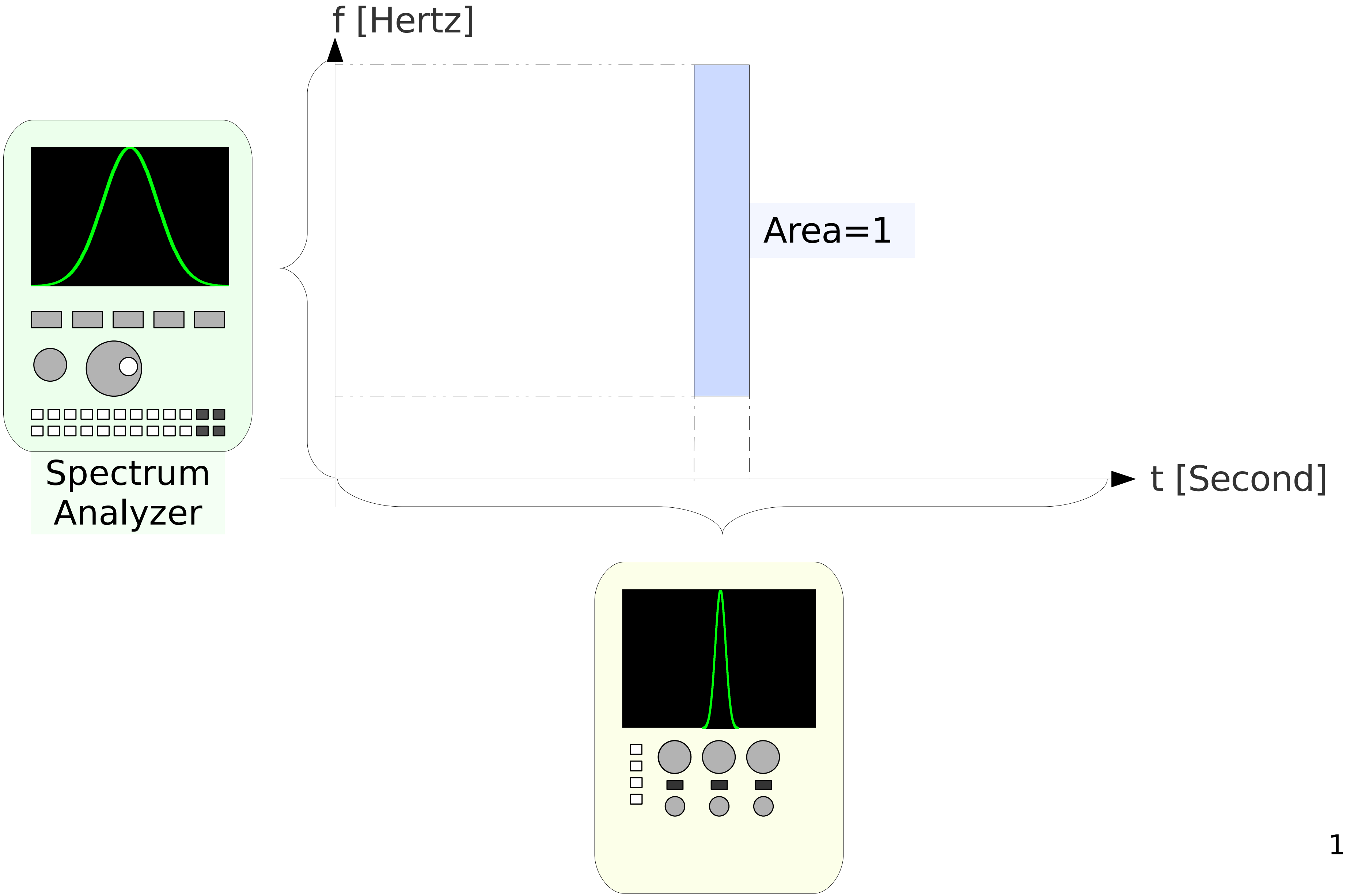
=> ONBs are adequate

- Orthonormal bases (ONBs) are the natural choice for transmission signal sets
- All ONBs are optimally robust w.r.t white noise
- Typical ONBs are robust w.r.t. (mildly) nonlinear distortions
- Typical ONBs are robust w.r.t. quantization

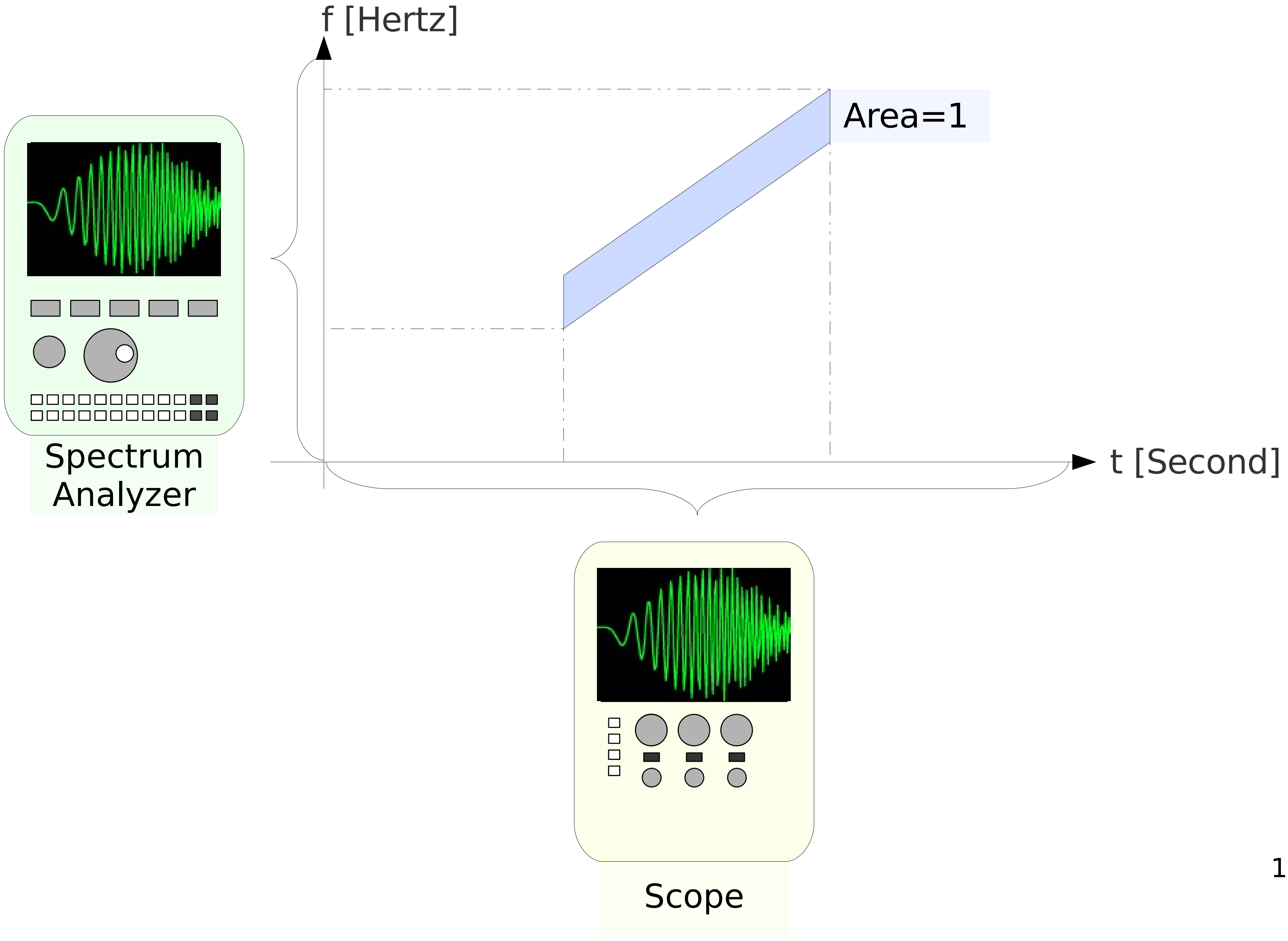
Uncertainty Principle



Example: WH Cell

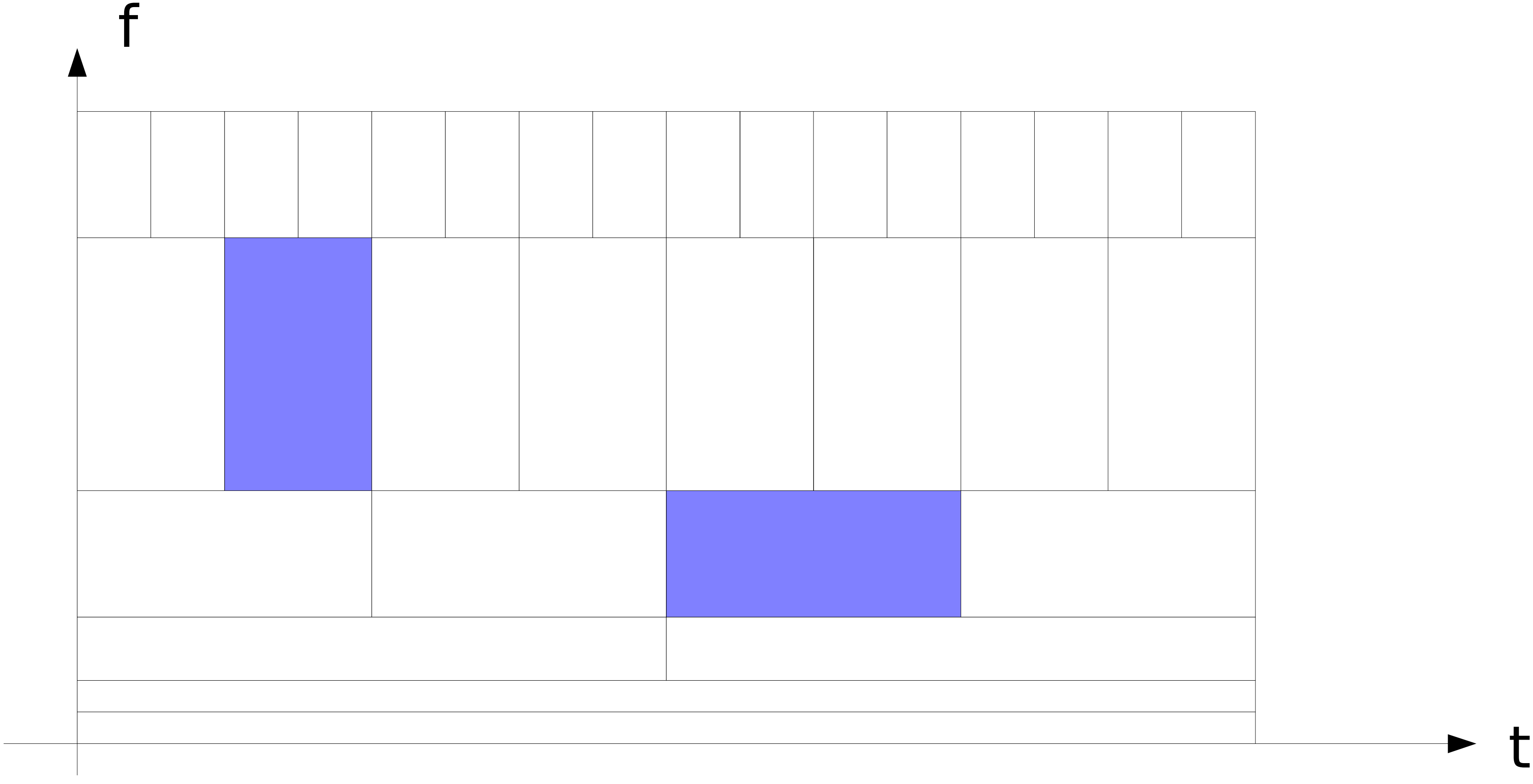


Example: WH Cell



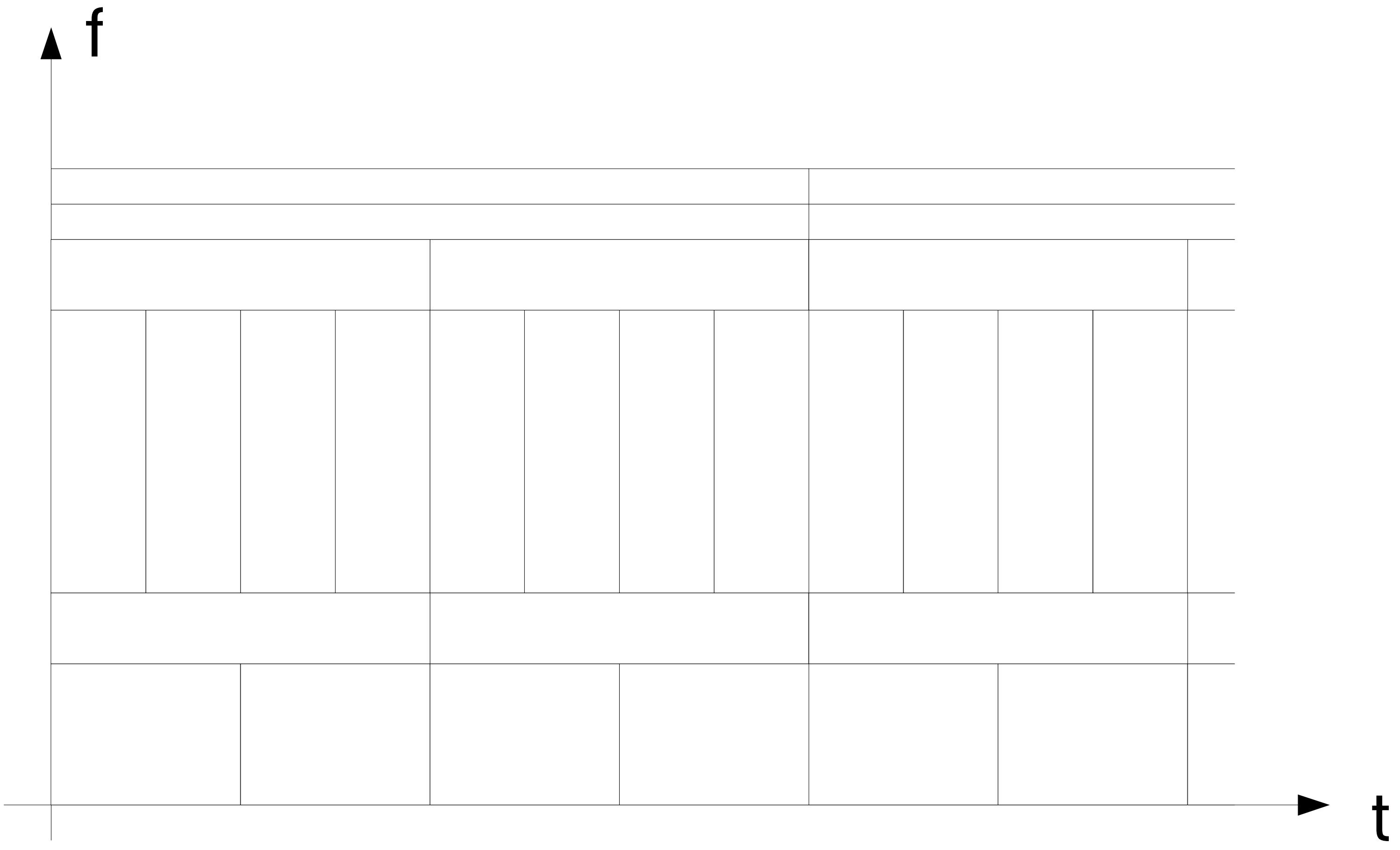
Wavelet (Constant Q) Tiling

- Wavelet tiling is mismatched to the LTI channel in the sense of perturbation stability / ease of equalization



Wavelet Packets

- Requires channel adaptivity of transmission base => complicated equalization, no feasible multiuser policy



DMT/OFDM = Rectangular Tilings

- OFDM = orthogonal frequency division multiplex
- OFDMA = orthogonal frequency division multiple acces

$$x(n) = \sum_{k=0}^{N} c_k \exp\left(j\, 2\pi \frac{kn}{N} \right), \quad x \in \mathbb{C}^N$$

- DMT = discret multitone modulation (base-band version of OFDM): N/2 complex coefficients mapped on N reals

$$x(n) = \sum_{k=0}^{N} c_k \exp\left(j\, 2\pi \frac{kn}{N} \right), \quad x \in \mathbb{R}^N$$

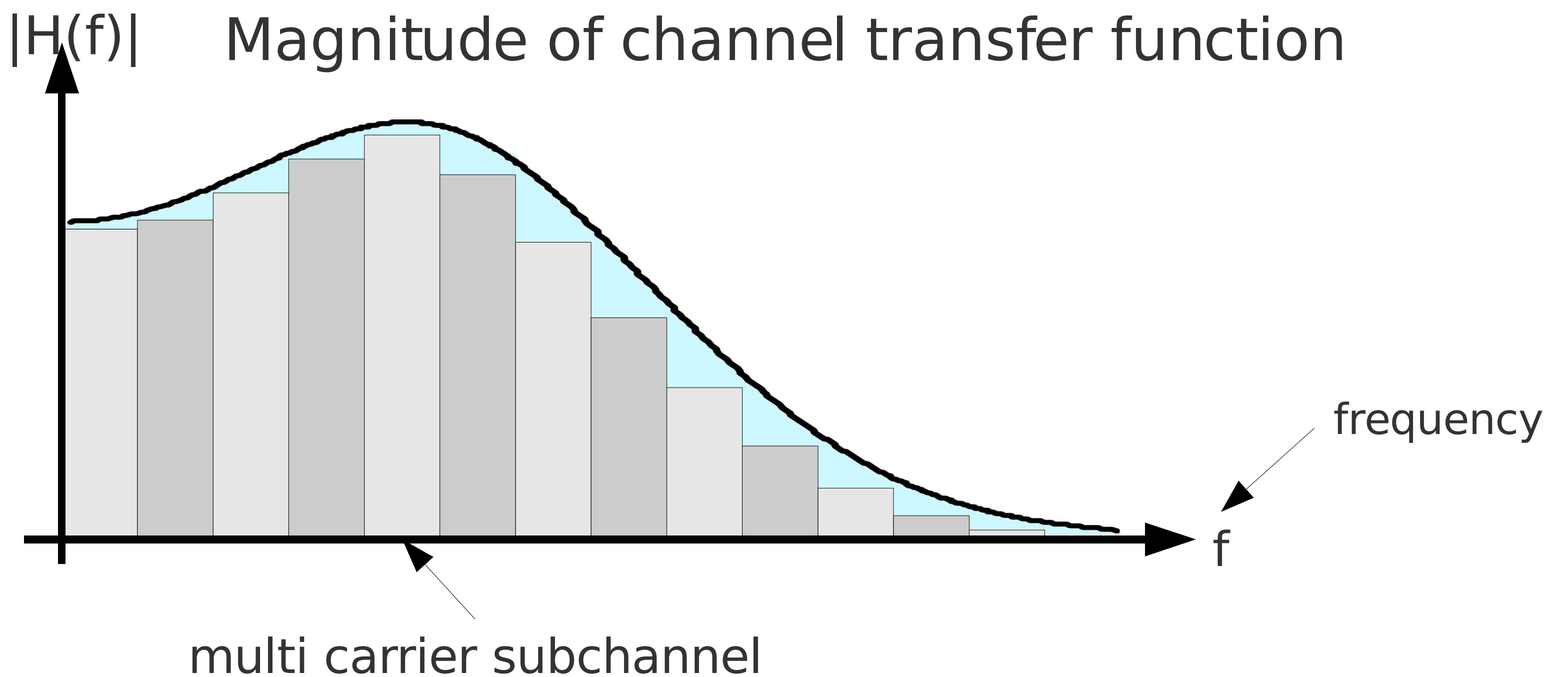
$c_{N-k} = \overline{c_k}$ conjugate symmetric extension

Approximation Aspect

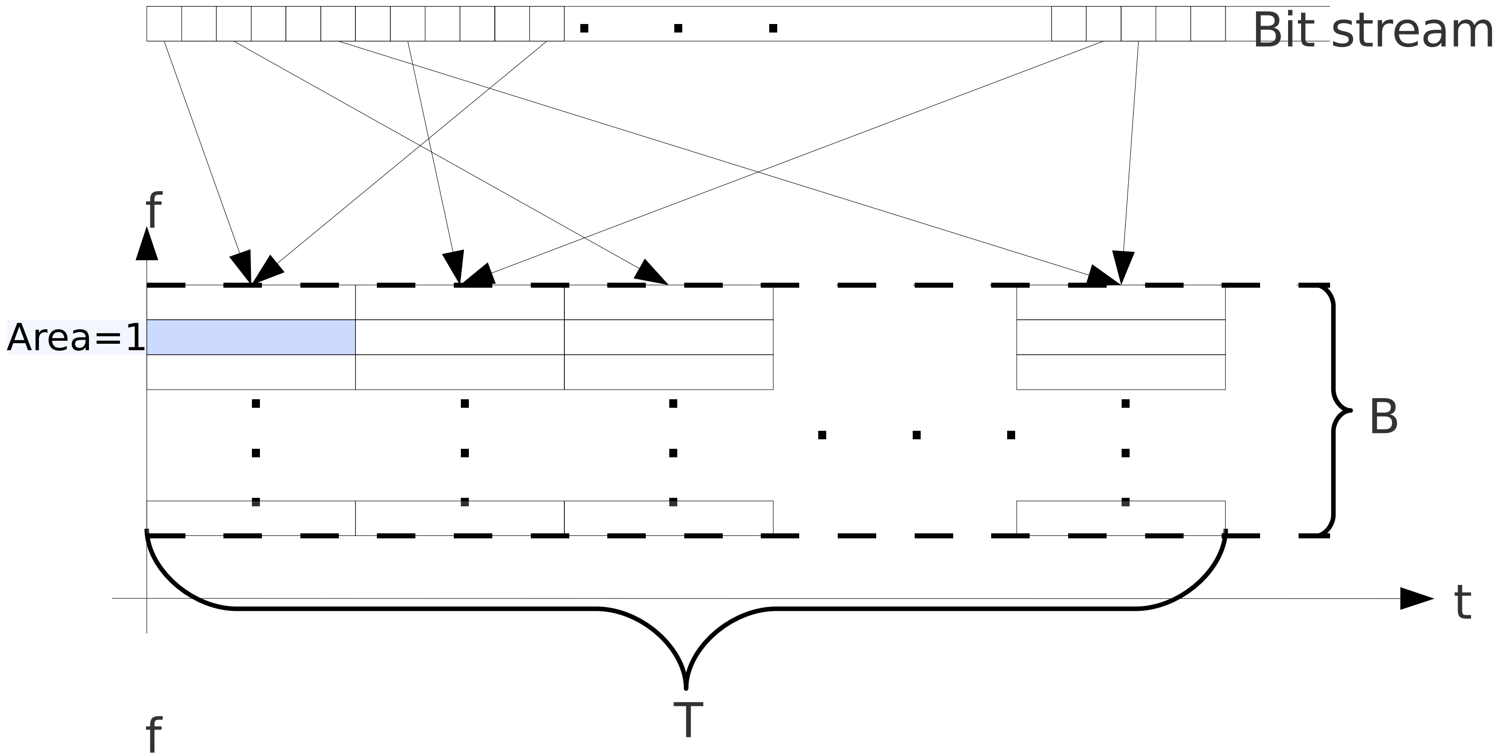
Shannon Capacity for LTI channels:

$$C = \int ld(1 + SNR(f))\,df$$

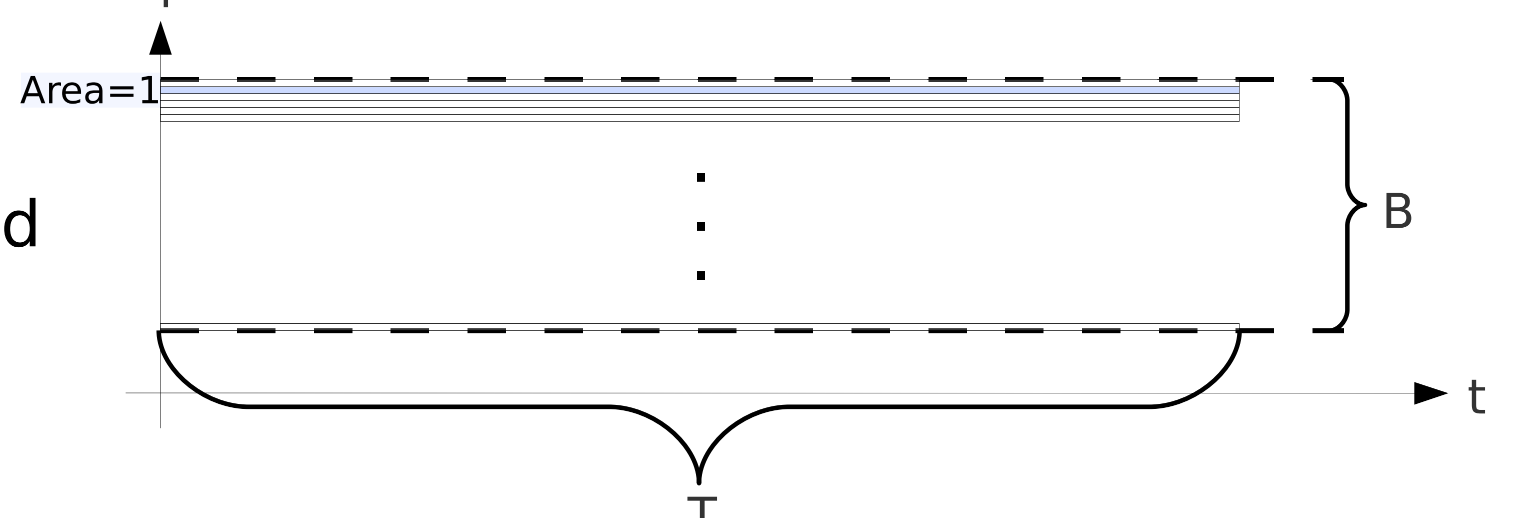
MCM can be interpreted as some sort of approximation implementing Shannon's theorem:



Where to Spend the Latency ?

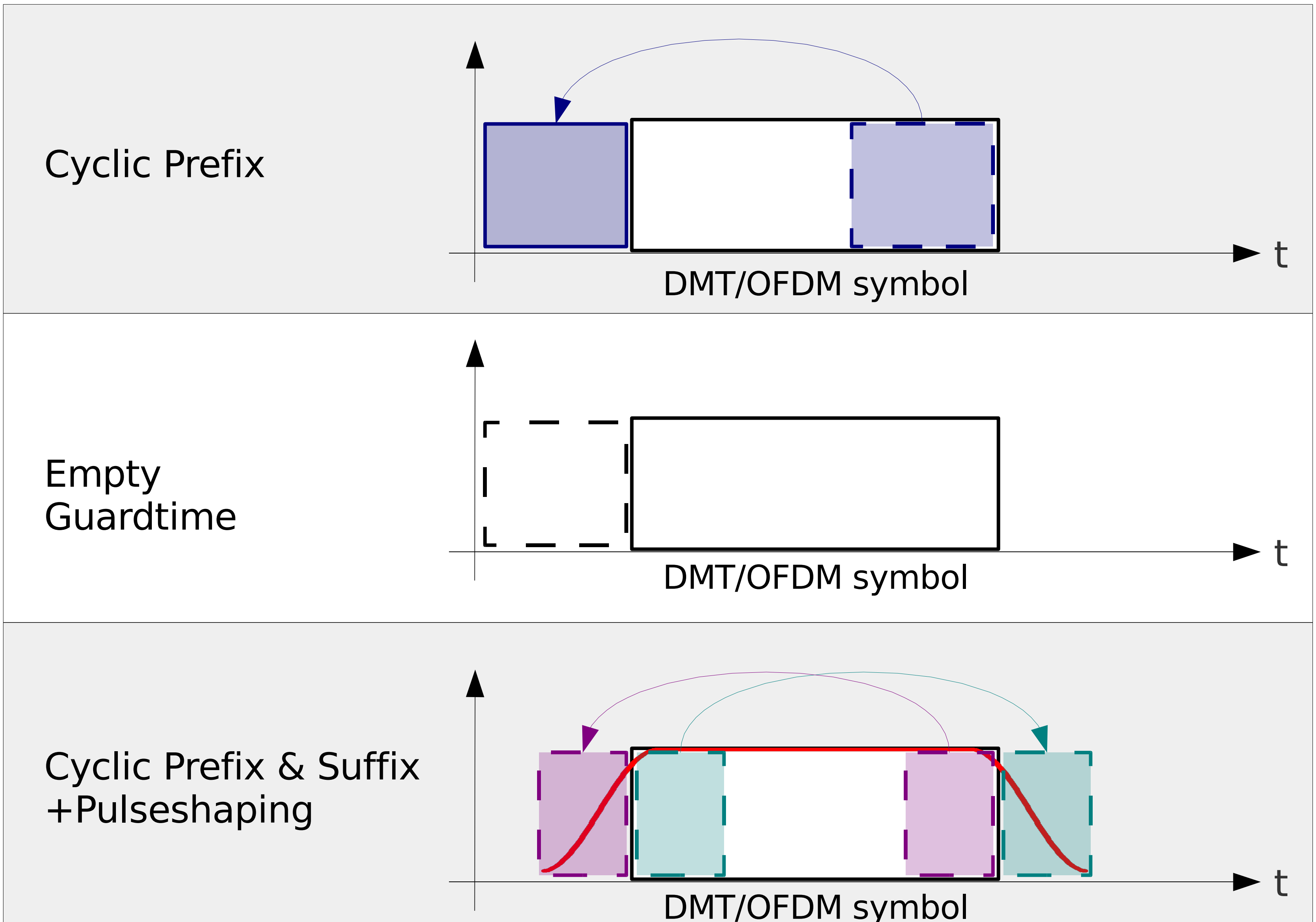


Coded OFDM



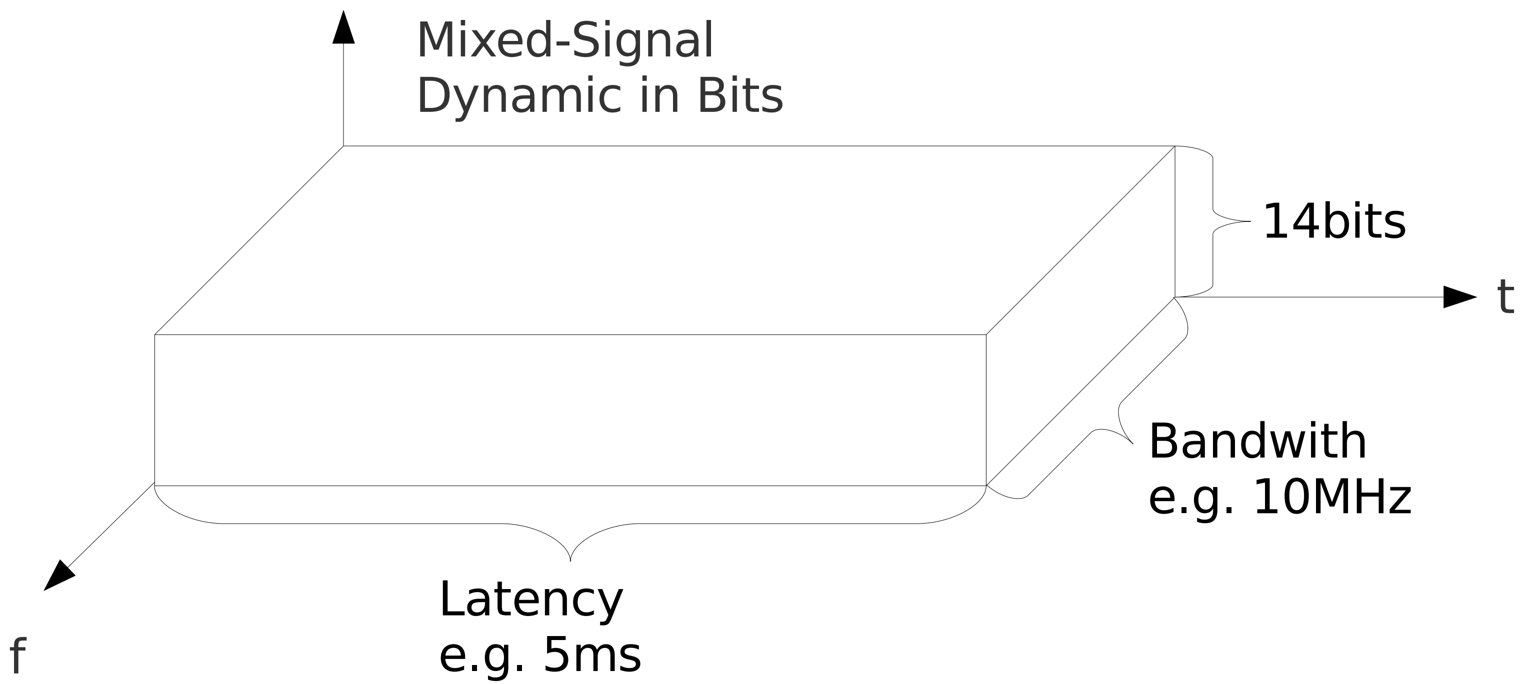
Uncoded OFDM

Reduction of ISI: Guard Interval



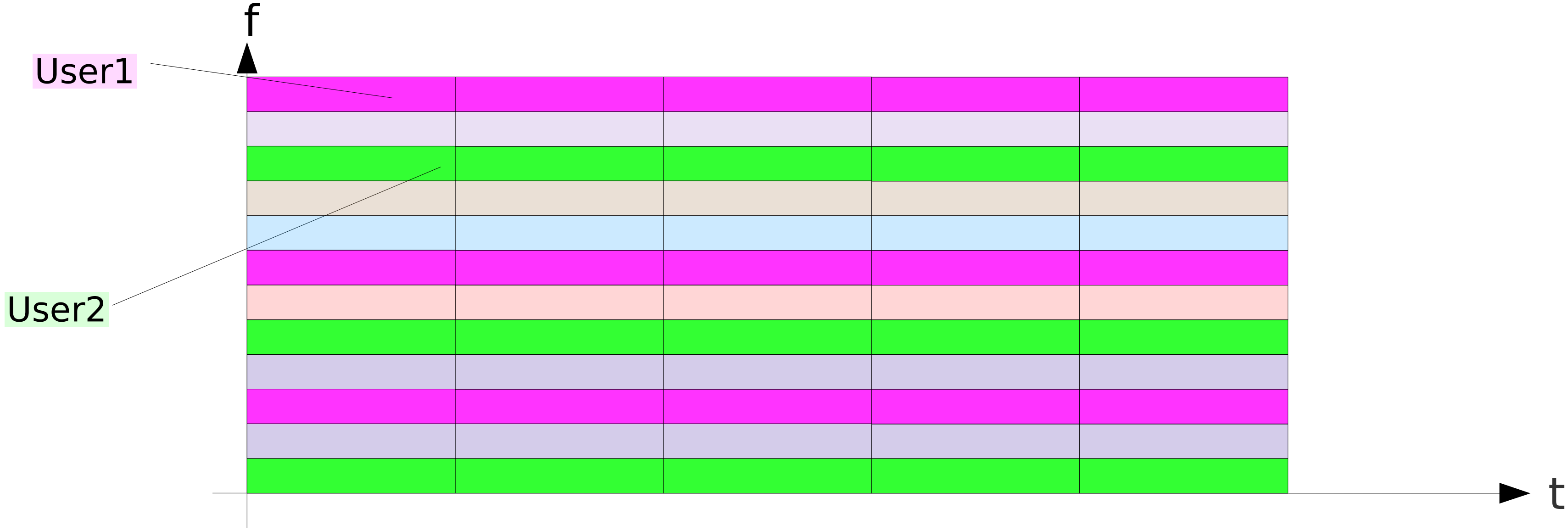
Quantization Aspect

- The classical Hilbert space methods underlying Shannon do not model the whole signal processing chain
- The transmit signal is confined to a convex manifold, a „message cuboid“:



Multiuser Aspect

- Multicarrier Modulation offers wide flexibility to apply deterministic and randomized multiple access methods
- As such it maps the traditional Hilbert space MU-detection problem onto integer sequence/matrix design
- Example: Subchannelization of WIMAX

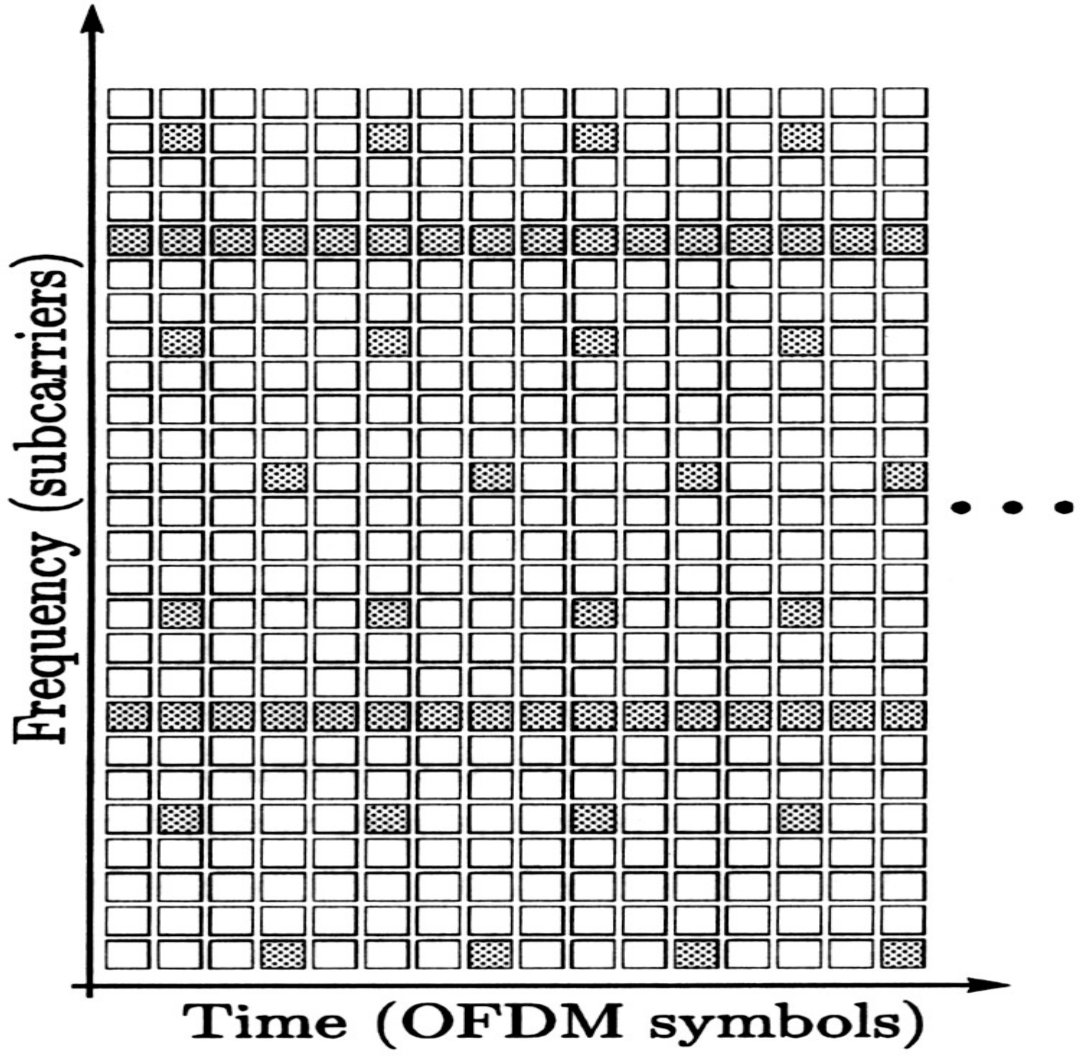


Channel Estimation

- *Wireline* systems (ADSL,VDSL2) estimate the stationary part of the channel during initialization phase
- Tracking of (very slowly) varying part of the inverse channel by frequency domain equalizer (one-tap stochastic gradient)
- *Wireless* systems (DVB-T, WIMAX) continuously estimate the channel via pilot symbols, i.e., OFDM symbols with known coefficients

Pilot Symbols

- Full subcarriers or lattice structures



Courtesy: M. Sandell 1996

Statistical Uncertainty Principle

- In the *estimation* of channel or noise you need at least 100 WH Cells to have an reliable estimate
- => there exists no *instantaneous* SNR
- => the average SNR can be obtained quite fast by averaging over frequency bins
- => the SNR/bin takes 100 time longer because you have to average over symbols
- for large scale crosstalk estimation problems one can used a layered architecture (divide-and-conquere)
  - 1.) binder group estimation
  - 2.) detailed crosstalk estimation within binder groups

(Semi)Blind Estimation of H

- Pilottones/symbols have a number of known problems:
  - overhead reduces user bit-rate
  - spectral zeros
- Blind methods: Based on imcomplete knowledge of output signal, often very poor convergence properties
- Known results: Exploit redundancy in OFDM transmission signal e.g. cyclic prefix
- Open: Exploit the evenly spread bit redundancy of FEC bits for channel estimation
- Open: Optimize bias/variance tradeoff for burst transmission

Autonomous Spectrum Management

- Centralized Downlink/Downstream spectrum management causes large overhead for SNMP management channels
- Open Problem: Optimized splitup between centralized and decentralized actions
- Incorporation of burst transmission rather than leased-line philosophy

WIMAX versus VDSL2

- Standardized versions for broadband internet access

- WIMAX(IEEE 802.16)
  - L-FFT = 128-2048
  - B= 1.25-20Mhz
  - Bitrate < 70Mbps
  - carrier-space =7.8Khz
  - Guardinterv. = CP
  - Bits/Tone <= 6
  - FEC: RS + (H)ARQ
  - Range = 50 km
  - User < 1000 (FDD/TDD)

- VDSL2 (ITU G.993.2)
  - L-FFT = 4096-8192
  - B=10-17Mhz
  - Bitrate < 150Mbps
  - carrier-space = 4/8Khz
  - Guardinterv. = CP+CS
  - Bits/Tone <=15
  - FEC: RS,TCM
  - Reach < 1km
  - User = 1

Conclusions

- Multicarrier Modulation is the predominant signal design for wireless and wireline communication due to
  - Diagonalization aspect: simplicity of equalization
  - Approximation aspect: simplicity of rate adaptation
  - Flexibility aspect: simplicity of spectrum management
  - Robustness aspect: nonlinearity in mixed signal domain
- Open issues of current interest :
  - semiblind channel estimation
  - autonomous spectrum management policy, in particular for burst transmission

THANK YOU