Multicarrier Modulation

Summer Academy at JUB

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Overview

- Digital Communication: Problem Setup
- Algebraic constraints for signal design:
  - Orthonormal, Biorthogonal, Overcomplete Systems
- Structural constraints for signal design:
  - Useful tilings of the time-frequency plane
- Latency Splitup: Coding versus Modulation
- Practical Examples: VDSL2, WIMAX
- Open Issues:
  - Semiblind channel estimation
  - Autonomous spectrum management
Problem Setup

Channel = Matrix-valued LTI operator

\[ y = Hx + n \]

input signal \( \in \mathbb{C}^{N \times M} \)

output signal \( \in \mathbb{C}^{N \times K} \)

Noise \( \in \mathbb{C}^{N \times K} \)
Pioneering MCM Literature

- Peled, Ruiz, Frequency domain data transmission using reduced computational complexity algorithms, 1980.
- Chow, Tu, Cioffi, A DMT transceiver System for HDSL Applications 1991.
- LeFloch, Alard, Berrou, Coded orthogonal frequency division multiplex 1995.
MC-Modem Building Blocks

Framer CRC → Scrambler & RS-Coder → Interleaver & Tone Ordering

Linear over GF(2)

QAM Mapper → Inv. FFT Add CP

Linear over C

DAC

Channel

ADC

Linear over C

TEQ Remove CP → FFT → FEQ & Slicer

Linear over GF(2)

De-Interleaver Tone Decoder → RS Decoder & Descrambler
Why Complex Numbers?

• Real-valued convolution operators are diagonalized by the complex valued Fourier transform

• Modern linear algebra allows to handle circulant matrices directly over the reals: Singular value decomposition (SVD)

• One needs two copies of the time-frequency plane: the sine-copy and the cosine-copy => slightly inconvenient

• This is how engineers handle the transmission of complex signals; their terminology is:
  - cosine-copy = „in-phase component“
  - sine-copy = „quadrature component“
Typical QAM Mappings

4QAM

8QAM

16QAM

32QAM
Signal Design Problem

- *Finite* amount of *discrete* data mapped onto signal within predefined TF-subspace (a large one typically):

\[ x(n) = \sum_k c_k g_k(n) \]

- Receiver needs to recover this information although the signal is subject to channel distortion and additive noise

- Divide-and-conquer: Split up the signal space into rank-one subspaces => series expansion of transmit signal

\[ \text{Bandwidth e.g. 10MHz} \]

\[ \text{Latency e.g. 5ms} \]
Orthonormal bases

- The standard setup for digital communication and information theory
- Transmission signal: \(x(n) = \sum_k c_k g_k(n)\)
- Receiver Detection: \(c_k = \langle x, g_k \rangle\)
- ONB condition: \(\langle g_k, g_l \rangle = \delta_{k,l} \quad \forall k, l\)
Biorthogonal bases

• In a Hilbert space setting unique recovery of coefficients in a series expansion requires Riesz bases rather than ONBs

• The coefficients are recovered with the help of a biorthogonal basis

• Transmission signal: \[ x(n) = \sum_k c_k g_k(n) \]

• Receiver Detection: \[ c_k = \langle x, f_k \rangle \]

• Biorthogonality Condition: \[ \langle g_k, f_l \rangle = \delta_{k,l} \quad \forall k, l \]

• (Beware: Communication Engineers use the phrase biorthogonality in a totally disjoint meaning)
Overcomplete Systems

- Consider a finite, discrete setting e.g. $\mathbb{C}^N$
- M vectors of length N < M
- These vectors are necessarily linear dependent => coefficients undetermined in a Hilbert space setting
- However, for finite alphabet coefficients recovery is in general possible
- Example: 3 Random Vectors on $\mathbb{R}^2$ and the binary coefficients with alphabet $\{0,1\}$
Overcomplete Systems (ctd.)

• Example: 3 Random Vectors on $\mathbb{R}^2$ and the binary coefficients with alphabet $\{0,1\}$

\[
\begin{align*}
&g_3 = x_{[100]} \\
&g_2 = x_{[010]} \\
&g_1 = x_{[001]} \\
&x_{[000]} \\
&x_{[001]} \
\end{align*}
\]

• Remark 1: Symmetrical choices lead to collisions
• Remark 2: Noise-free observation of at least one real number is *pure* mathematics
=> ONBs are adequate

- Orthonormal bases (ONBs) are the natural choice for transmission signal sets
- All ONBs are optimally robust w.r.t white noise
- Typical ONBs are robust w.r.t. (mildly) nonlinear distortions
- Typical ONBs are robust w.r.t. quantization
Uncertainty Principle

\[ f \text{ [Hertz]} \]

\[ t \text{ [Second]} \]

Area = 1
Example: WH Cell

Spectrum Analyzer

Area=1

\[ f \text{ [Hertz]} \]

\[ t \text{ [Second]} \]
Example: WH Cell

Spectrum Analyzer

Scope

\[ f [\text{Hertz}] \]

\[ t [\text{Second}] \]

Area = 1
Wavelet (Constant Q) Tiling

- Wavelet tiling is mismatched to the LTI channel in the sense of perturbation stability / ease of equalization.
Wavelet Packets

- Requires channel adaptivity of transmission base => complicated equalization, no feasible multiuser policy
DMT/OFDM = Rectangular Tilings

- OFDM = orthogonal frequency division multiplex
- OFDMA = orthogonal frequency division multiple access

\[ x(n) = \sum_{k=0}^{N} c_k \exp\left( j 2\pi \frac{kn}{N} \right), \quad x \in \mathbb{C}^N \]

- DMT = discrete multitone modulation (base-band version of OFDM): N/2 complex coefficients mapped on N reals

\[ x(n) = \sum_{k=0}^{N} c_k \exp\left( j 2\pi \frac{kn}{N} \right), \quad x \in \mathbb{R}^N \]

\[ c_{N-k} = \overline{c_k} \quad \text{conjugate symmetric extension} \]
Approximation Aspect

Shannon Capacity for LTI channels:

\[ C = \int \log(1 + SNR(f)) \, df \]

MCM can be interpreted as some sort of approximation implementing Shannon's theorem:
Where to Spend the Latency?

Coded OFDM

Uncoded OFDM

Bit stream

Area=1
Reduction of ISI: Guard Interval

Cyclic Prefix

Empty Guardtime

Cyclic Prefix & Suffix + Pulseshaping
Quantization Aspect

- The classical Hilbert space methods underlying Shannon do not model the whole signal processing chain.
- The transmit signal is confined to a convex manifold, a „message cuboid“:

![Diagram showing mixed-signal dynamic in bits, latency, bandwidth, and time.](image)

- Bandwidth: e.g. 10MHz
- Latency: e.g. 5ms
- Mixed-Signal Dynamic in Bits: 14bits
Multiuser Aspect

- Multicarrier Modulation offers wide flexibility to apply deterministic and randomized multiple access methods.
- As such it maps the traditional Hilbert space MU-detection problem onto integer sequence/matrix design.
- Example: Subchannelization of WIMAX.
Channel Estimation

- **Wireline** systems (ADSL, VDSL2) estimate the stationary part of the channel during initialization phase.
- Tracking of (very slowly) varying part of the inverse channel by frequency domain equalizer (one-tap stochastic gradient).
- **Wireless** systems (DVB-T, WIMAX) continuously estimate the channel via pilot symbols, i.e., OFDM symbols with known coefficients.
Pilot Symbols

- Full subcarriers or lattice structures

Courtesy: M. Sandell
1996
Statistical Uncertainty Principle

- In the estimation of channel or noise you need at least 100 WH Cells to have a reliable estimate
- => there exists no instantaneous SNR
- => the average SNR can be obtained quite fast by averaging over frequency bins
- => the SNR/bin takes 100 time longer because you have to average over symbols
- for large scale crosstalk estimation problems one can use a layered architecture (divide-and-conquer)
  - 1.) binder group estimation
  - 2.) detailed crosstalk estimation within binder groups
(Semi)Blind Estimation of $H$

- Pilot tones/symbols have a number of known problems:
  - overhead reduces user bit-rate
  - spectral zeros

- Blind methods: Based on incomplete knowledge of output signal, often very poor convergence properties

- Known results: Exploit redundancy in OFDM transmission signal e.g. cyclic prefix

- Open: Exploit the evenly spread bit redundancy of FEC bits for channel estimation

- Open: Optimize bias/variance tradeoff for burst transmission
Autonomous Spectrum Management

- Centralized Downlink/Downstream spectrum management causes large overhead for SNMP management channels
- Open Problem: Optimized splitup between centralized and decentralized actions
- Incorporation of burst transmission rather than leased-line philosophy
WIMAX versus VDSL2

- Standardized versions for broadband internet access

- **WIMAX (IEEE 802.16)**
  - L-FFT = 128-2048
  - B = 1.25-20Mhz
  - Bitrate < 70Mbps
  - carrier-space = 7.8Khz
  - Guardinterv. = CP
  - Bits/Tone <= 6
  - FEC: RS + (H)ARQ
  - Range = 50 km
  - User < 1000 (FDD/TDD)

- **VDSL2 (ITU G.993.2)**
  - L-FFT = 4096-8192
  - B = 10-17Mhz
  - Bitrate < 150Mbps
  - carrier-space = 4/8Khz
  - Guardinterv. = CP+CS
  - Bits/Tone <= 15
  - FEC: RS, TCM
  - Reach < 1km
  - User = 1
Conclusions

• Multicarrier Modulation is the predominant signal design for wireless and wireline communication due to
  – Diagonalization aspect: simplicity of equalization
  – Approximation aspect: simplicity of rate adaptation
  – Flexibility aspect: simplicity of spectrum management
  – Robustness aspect: nonlinearity in mixed signal domain

• Open issues of current interest:
  – semiblind channel estimation
  – autonomous spectrum management policy, in particular for burst transmission
THANK YOU