Equalizer Variants for Discrete Multi-Tone (DMT) Modulation

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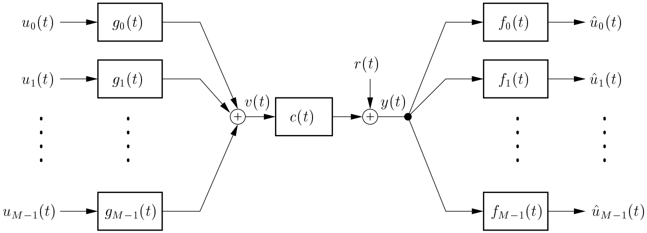
Never stop thinking

Outline

Introduction to Multi-Carrier (MC) Systems



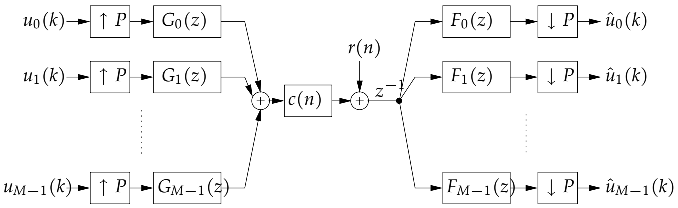
- split a wide-band, frequency-selective channel into large number of small-band, assumed flat subchannels for simplified equalization
- robust to small-band interferers and impulse noise
- optimal power and bit allocation according to channel characteristics → best suited for DSM

A Short History on MC Systems

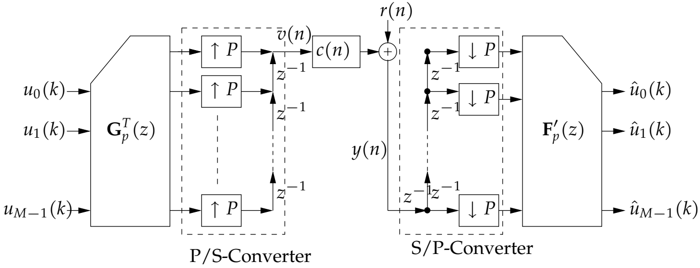
- **algorithms** known since the 1960's
- first Multi-Carrier modems in **conventional frequency-multiplex technology**: analog filters for band separation, poor bandwidth efficiency
- **staggered QAM (SQAM)** systems: close to 100 percent bandwidth efficiency, overlap at $f_{3dB} \rightarrow$ constant sum spectrum, interference to the direct neighbors, orthogonality due to alternating in-phase and quadrature modulation, less than 20 subcarriers
- large number of subcarriers with **Orthogonal Multi-Carrier (OMC)**: band pass filters with si-like spectrum, complex modulated filterbank with rectangular prototype filter $\rightarrow$ **OFDM, DMT**
- **Discrete Wavelet Multi-Tone (DWMT)**: cosine-modulated filterbanks, inter-channel interference reduced to a minimum

The Multi-Carrier Transmission Scheme

- discrete MC system, $P \geq M$



- equivalent polyphase representation of the basis filters

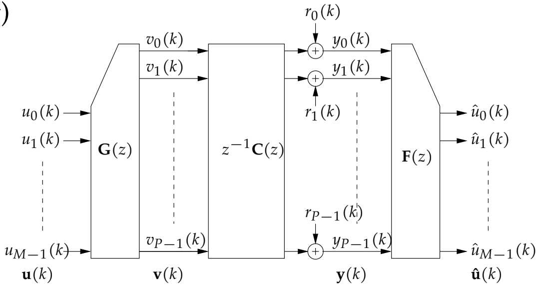


The MC Transmission Scheme (cont'd)

→ Combining polyphase representation of channel impulse response and P/S, S/P converters to circular polyphase matrix $\mathbf{C}(z)$ plus additional delay

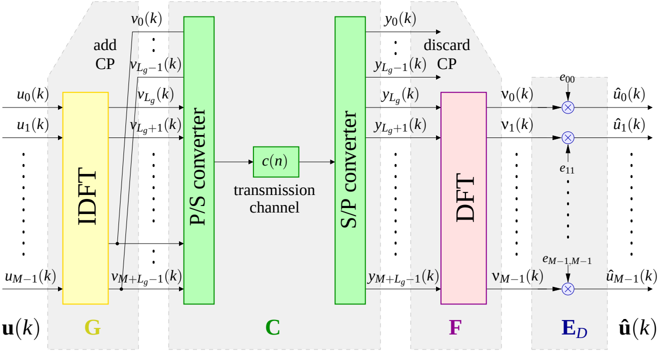
$$\mathbf{C}(z) = \begin{bmatrix} C_0(z) & z^{-1}C_{P-1}(z) & \cdots & z^{-1}C_1(z) \\ C_1(z) & C_0(z) & & z^{-1}C_2(z) \\ \vdots & & \ddots & \vdots \\ C_{P-1}(z) & C_{P-2}(z) & \cdots & C_0(z) \end{bmatrix}$$

→ Introducing redundancy: $P = M + L$ with $L \geq 0$ ($L = 0 \rightarrow$ critical sampling)



Discrete Multi-Tone (DMT) Modulation

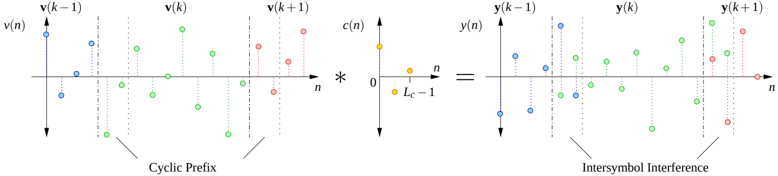
- Multi-Carrier transmission scheme based on IDFT/DFT → basic structure similar to Orthogonal Frequency Division Multiplexing (OFDM) for wireless transmission



- application examples: ADSLx, VDSLx, WLAN, 3G, DVB-T, DAB, UWB, Powerline, ...

The Cyclic Prefix (CP)

- repeat last L_g samples at the beginning of the symbol



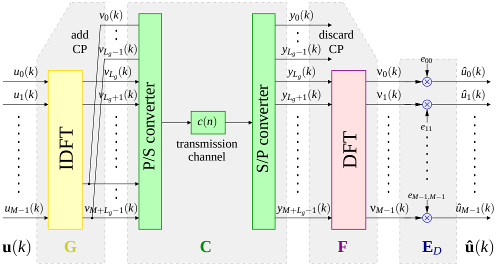
- Perfect equalization in a noise-free environment, if

$$L_g \geq L_c - 1$$

(L_g - length of cyclic prefix, L_c - length of channel impulse response)

- Strong Intersymbol and Intercarrier Interference (ISI/ICI) if above criterion is not fulfilled

Transfer function of a DMT system



→ transfer function in polyphase and block matrix notation:

$$\hat{\mathbf{U}}(z) = \mathbf{E}(z) \cdot \mathbf{F}(z) \cdot z^{-1} \cdot \mathbf{C}(z) \cdot \mathbf{G}(z) \cdot \mathbf{U}(z)$$

$$\hat{\mathbf{u}}(k) = \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C} \cdot \mathbf{G} \cdot \mathbf{u}^{(n)}(k-1)$$

(n – number of interfering symbols)

Inter-Symbol/Inter-Carrier Interference

$$\hat{\mathbf{u}}(k) = \underbrace{\mathbf{E} \cdot \overbrace{\mathbf{F} \cdot \mathbf{C} \cdot \mathbf{G}}^{\mathbf{H}}}_{\mathbf{T}} \cdot \mathbf{u}^{(n)}(k-1)$$

Sufficient CP: $L_g \geq L_c - 1$

- $n = 1 \rightarrow$ no ISI
- cyclic prefix $\rightarrow$ sequential convolution with channel impulse response becomes virtually cyclic $\rightarrow$ no ICI

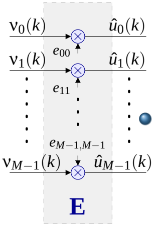
Insufficient CP: $L_g < L_c - 1$

- $n > 1 \rightarrow$ (severe) ISI
- no longer cyclic convolution $\rightarrow$ (severe) ICI

ISI/ICI cont'd

Equalizer Matrix for Sufficient CP

$$\mathbf{T} = \mathbf{E} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{D} \stackrel{!}{=} \mathbf{I}$$

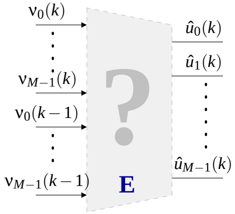


- $\mathbf{D}$ diagonal matrix, solution $\mathbf{E} = \mathbf{D}^{-1}$ with $e_{ii} = 1/C(e^{j2\pi i/M})$
- Efficient equalization with only one complex multiplication per tone

Equalizer Matrix for Insufficient CP

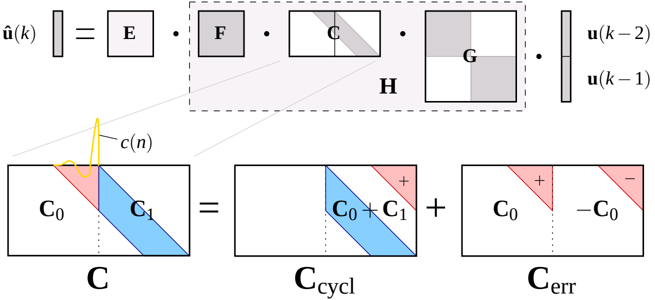
$$\mathbf{T} = \mathbf{E} \cdot \mathbf{H} \stackrel{!}{=} [\underbrace{\mathbf{0} \cdots \mathbf{0}\, \mathbf{I}\, \mathbf{0} \cdots \mathbf{0}}_{n}]$$

- Solution only if $\mathbf{E}$ takes neighboring symbols into account
- $\mathbf{E}$ very big and no sparse structure



Extracting ISI/ICI Part from Channel Matrix C

- Assumption: no Cyclic Prefix, $L_c \leq M$, no pre-cursor $\rightarrow$ ISI from the preceding symbol



Extracting ISI/ICI Part (cont'd)

⇒ $\mathbf{C}$ can be split into ideal, cyclic part $\mathbf{C}_{\text{cycl}}$ and ISI/ICI error part $\mathbf{C}_{\text{err}}$

$$\mathbf{C} = [\mathbf{C}_0 \quad \mathbf{C}_1] = \underbrace{[\mathbf{0}_M \quad (\mathbf{C}_0 + \mathbf{C}_1)]}_{\mathbf{C}_{\text{cycl}}} + \underbrace{[\mathbf{C}_0 \quad (-\mathbf{C}_0)]}_{\mathbf{C}_{\text{err}}}$$

⇒ Substitution into transfer function

$$\mathbf{E} \cdot \mathbf{H} = \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C} \cdot \mathbf{G} = \mathbf{E} \cdot \mathbf{F} \cdot (\mathbf{C}_{\text{cycl}} + \mathbf{C}_{\text{err}}) \cdot \mathbf{G} \stackrel{!}{=} [\mathbf{0} \quad \mathbf{I}]$$

Extracting ISI/ICI Part (cont'd)

⇒ With $\mathbf{C}_{\text{cycl,red}} = \mathbf{C}_0 + \mathbf{C}_1$ and $\mathbf{G}'$ as a diagonal block in $\mathbf{G}$, separation into two sub-systems possible

$$\mathbf{I}: \quad \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C}_{\text{cycl,red}} \cdot \mathbf{G}' \stackrel{!}{=} \mathbf{I}$$

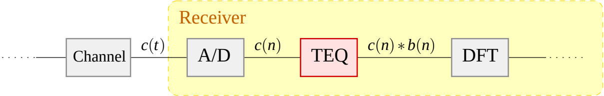
$$\mathbf{II}: \quad \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C}_{\text{err}} \cdot \mathbf{G} \stackrel{!}{=} \begin{bmatrix} \mathbf{0} & \mathbf{0} \end{bmatrix}$$

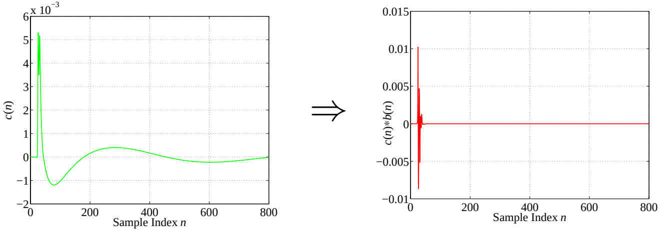
→ Equation system **I** describes an ideal, distortion-free DMT system

→ Equation system **II** eliminates ISI/ICI caused by $\mathbf{C}_{\text{err}}$

The Time-Domain Equalizer (TEQ)

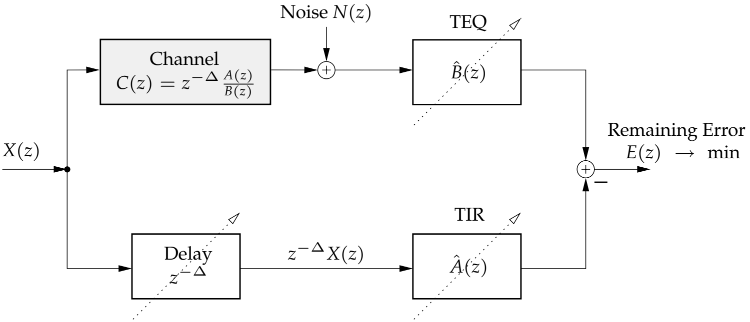
- FIR filter applied before receiver FFT to shorten effective length of the channel impulse response





- filters at sampling rate → complexity!
- mandatory for at least ADSL, determines performance for shorter loops
- optimal adaptation of the TEQ coefficients in the SNR sense too costly

TEQ - MMSE Method



- adopt delay Δ, **TEQ** filter $\hat{B}(z)$ and **Target Impulse Response (TIR)** $\hat{A}(z)$ to **minimize** the **minimum mean square error** between the two branches $\text{MSE} = E[|e(k)|^2]$ with $e(k) \circ\!\!-\!\!\bullet\ E(z)$
- first introduced by Falconer and Magee (1973) for channel shortening of a ML receiver

TEQ - MMSE, Variants

- Chow and Cioffi (1992) first applied for MC, to prevent trivial solution **Unit-Tap Constraint** (**UTC**) was introduced:

$$\hat{\mathbf{a}}^T \mathbf{e}_i = 1 \quad \text{with } \hat{\mathbf{a}} = [\hat{a}_0\, \hat{a}_1 \cdots \hat{a}_{L_{\text{TIR}}-1}]^T$$

- UTC requires search over all i, matrix inversions and multiplications
- Al-Dhahir and Cioffi (1996) showed that the **Unit-Energy Constraint** (**UEC**):

$$\hat{\mathbf{a}}^T \hat{\mathbf{a}} = 1$$

- always leads to equal or smaller MSE than UTC, no search over all i, no matrix inversion, but eigenvalue calculation required

$\Rightarrow$ high complexity due to extensive matrix operations

$\Rightarrow$ Lee, Chow and Cioffi (1995) proposed circulant correlation matrices $\rightarrow$ matrix operations using DFT/IDFT, but $L_{\text{TIR}} \geq L_{\text{TEQ}}$

TEQ - MMSE, Adaptation Methods

- Falconer, Magee, 1973: proposed **LMS** for adaptation, low complexity but slow
- Chow, Cioffi, Bingham, 1993: **Frequency-Domain LMS** and **Frequency Domain Division**, alternating between FD and TD, still slow convergence
- Melsa, Younce, Rohrs, 1996: **ARMA modelling** of the channel, **LS** or **RLS** solution, efficient variant: translation to **2-channel AR** model and solved with **Levinson-Wiggins-Robinson** (**LWR**) algorithm
- Wang, Adali, 1999: solve for MSE completely in the frequency domain, **weight factor for each tone** to exclude unused tones from optimization

TEQ - MSSNR Method

- effective channel impulse response incl. TEQ $c_{\text{eff}}(n) = c(n) * h_{\text{TEQ}}(n) \rightarrow \mathbf{c}_{\text{eff}} = [c_0\, c_1 \cdot c_{L_c + L_{\text{TEQ}} - 1}]^T$ split into kernel segment $\mathbf{c}_{\text{win}}$ and pre and post cursor $\mathbf{c}_{\text{wall}}$
- minimize ISI contributing energy $\mathbf{c}_{\text{wall}}^T \mathbf{c}_{\text{wall}}$, i.e. maximize the **Shortening SNR**

$$\text{SSNR}\,[\text{dB}] = 10 \cdot \log_{10} \frac{\mathbf{c}_{\text{win}}^T \mathbf{c}_{\text{win}}}{\mathbf{c}_{\text{wall}}^T \mathbf{c}_{\text{wall}}}$$

- originally proposed by Melsa, Younce and Rohrs (1996), optimal solution
- optimal solution, involves Cholesky decomposition, matrix inversion, Eigenvalue calculation
- not suited for application, serves as a reference

TEQ - MSSNR, DCC and DCM

- based on **Divide-and-Conquer** Ansatz, introduced by Lu, Clark, Arslan and Evans (2000)
- **Divide-and-Conquer**: TEQ filter of length L_{TEQ} is factorized into $L_{\text{TEQ}} - 1$ filters $\mathbf{w}_i$ of length 2

$$\mathbf{w}_i = [1, g_i]^T$$

- g_i's iteratively optimized using two different approaches: **Divide-and-Conquer by Cancellation** (**DCC**) and **Divide-and-Conquer by Minimization** (**DCM**)
- nearly optimal solution
- computationally efficient, suitable for practical implementation
- odd behavior: energy concentration in first TEQ filter taps

TEQ - MGSNR Method

- bit rate of a DMT system:

$$b_{\text{DMT}} = \sum_{i=0}^{N-1} \log_2 \left(1 + \frac{\text{SNR}_i}{\Gamma}\right) \quad \rightarrow \quad b_{\text{DMT}} = N \log_2 \left(1 + \frac{\overline{\text{SNR}}}{\Gamma}\right)$$

$$\text{with} \quad \overline{\text{SNR}} = \Gamma \left(\left[\prod_{i=0}^{N-1} \left(1 + \frac{\text{SNR}_i}{\Gamma}\right) \right]^{\frac{1}{N}} - 1 \right) \approx \text{GSNR}$$

- maximize geometric mean GSNR of the SNR_i over all used tones
- Al-Dhahir and Cioffi (1996) proposed suboptimal solution (due to some approximations)
- Henkel, and Kessler (2000) take external noise into account, optimal TIR may be longer than cyclic prefix
- Arslan, Evans and Kaiei (2001) derive more efficient Minimum-ISI method from nonlinear optimization problem

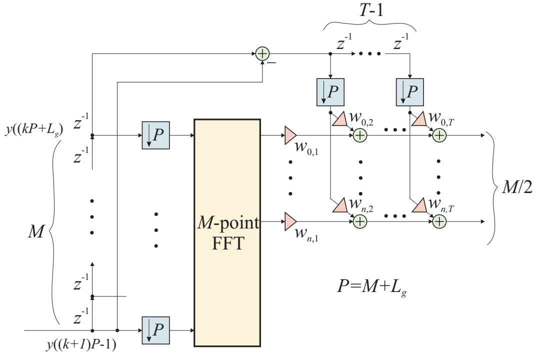
$\Rightarrow$ all MGSNR methods far too complex for implementation

TEQ - "Recent" Methods

- Arslan, Lu, Clark, Evans, 2001: (Modified) Matrix pencil design method
- Farhang-Boroujeny, Ding, 2001: Eigen approach
- Martin, Johnson, Ding, Evans, 2003: Symmetric maximum shortening SNR

$\Rightarrow$ for further information have a look at:
http://users.ece.utexas.edu/ bevans/projects/adsl/dmtteq/dmtteq.html

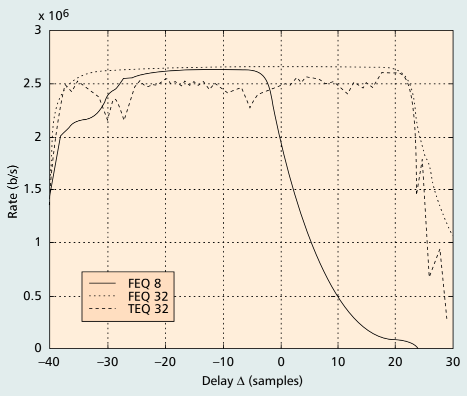
The Per-Tone Equalizer [Van Acker et al]



- transfer T-tap TEQ operation to FD
  - → requires T-times sliding FFT
  - → efficient realization with single FFT and subsequent "difference" terms
- separate T-tap FEQ for each tone
  - → increased memory requirements
- similar complexity compared to TEQ

PTEQ (cont'd)

- N-times more coefficients to be initialized → "tone grouping"
- efficient RLS-based method initialization method proposed, LMS for updates during runtime
- larger T always improves performance
- insensitive to delay parameter



Extracting ISI/ICI Part (cont'd)

$\Rightarrow$ With $\mathbf{C}_{\text{cycl,red}} = \mathbf{C}_0 + \mathbf{C}_1$ and $\mathbf{G}'$ as a diagonal block in $\mathbf{G}$, separation into two sub-systems possible

$$\textbf{I:} \quad \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C}_{\text{cycl,red}} \cdot \mathbf{G}' \stackrel{!}{=} \mathbf{I}$$

$$\textbf{II:} \quad \mathbf{E} \cdot \mathbf{F} \cdot \mathbf{C}_{\text{err}} \cdot \mathbf{G} \stackrel{!}{=} [\mathbf{0} \quad \mathbf{0}]$$

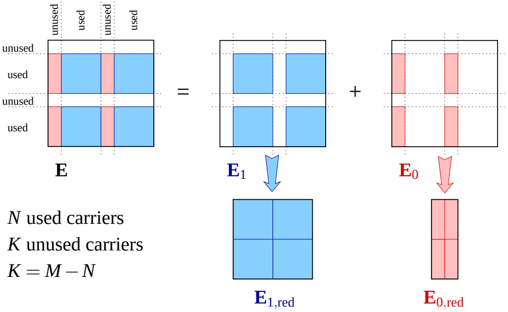
$\rightarrow$ Equation system **I** describes an ideal, distortion-free DMT system

$\rightarrow$ Equation system **II** eliminates ISI/ICI caused by $\mathbf{C}_{\text{err}}$

Decomposing Equalizer Matrix E

- Under assumption that K tones not used decomposition of equalizer matrix into

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_0$$



N used carriers
K unused carriers
$K = M - N$

$\longrightarrow$ $\mathbf{E}_0$ contains K columns of unused tone output samples

$\longrightarrow$ $\mathbf{E}_1$ contains $N = M - K$ columns of used tone output samples

Perfect Equalization Condition

⇒ After elimination of zero rows and columns equation system **I** independent from $\mathbf{E}_0$

$$\mathbf{E}_{1,\mathrm{red}} \cdot \underbrace{\mathbf{F}_{1,\mathrm{red}} \cdot \mathbf{C}_{\mathrm{cycl,red}} \cdot \mathbf{G}'_{\mathrm{red}}}_{\mathbf{D}_{\mathrm{red}}} \stackrel{!}{=} \mathbf{I}_N$$

- solution for **I** with $\mathbf{E}_{1,\mathrm{red}} = \mathbf{D}_{\mathrm{red}}^{-1} \rightarrow$ deg. of freedom in $\mathbf{E}_{0,\mathrm{red}}$ used for solving **II** !

⇒ Solution independent from channel frequency response at the unused carrier positions!

Perfect Equalization Condition (cont'd)

- Solution for **II** exists if

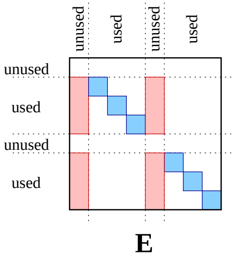
$$K + L_g \geq L_c - 1$$

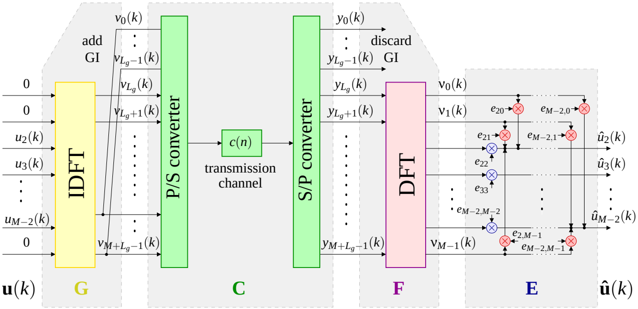
$\Rightarrow$ combination of TD and FD redundancy $\rightarrow$ may be arbitrarily distributed

- Special cases:
  - $\rightarrow$ $K = 0$: Traditional DMT/OFDM without usage of FD redundancy
  - $\rightarrow$ $L_g = 0$: No cyclic prefix, but symbol-separate, perfect equalization!

Generalized DMT [Trautmann et al]

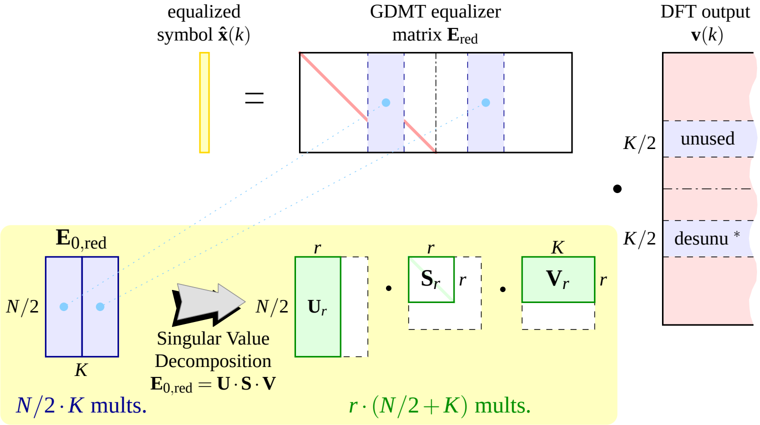
- Transmitter similar to DMT, Receiver with slightly extended Equalizer → Sparse Matrix **E**
- Redundancy may be arbitrarily distributed to either time-domain (length of cyclic prefix), or frequency domain (number of unused subcarriers)





Complexity Reduction for GDMT?

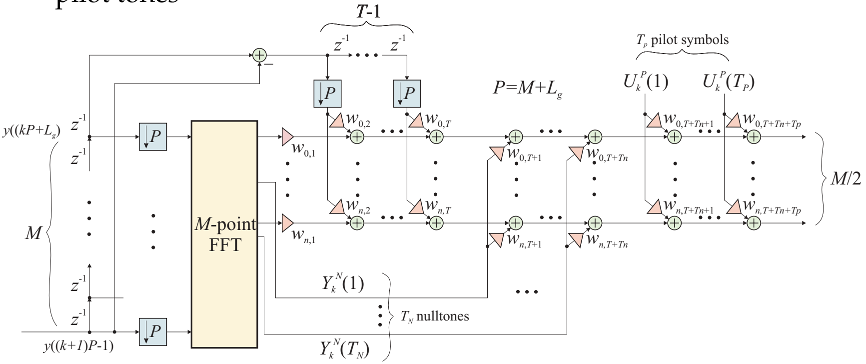
- SVD decomposition
- Only meaningful for the branches from the measurement tones



- Complexity reduced to $\frac{r \cdot (N/2+K)}{N/2 \cdot K}$ (tends to r/K for $N \gg K$)

The Extended PTEQ [Vanbleu et al]

- extra FD redundancy with T_N nulltones = T_Z zero tones + T_P pilot tones

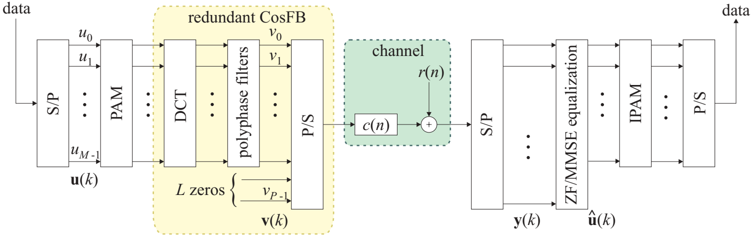


- perfect equalization for IIR channels $C(z) = \frac{A(z)}{B(z)} = \frac{\sum_{l=0}^{L_A-1} a_l z^{-l}}{\sum_{l=0}^{L_B-1} b_l z^{-l}}$ if

$$T_N + L_g \geq L_A - 1 \quad \text{and} \quad T \geq L_B$$

Other Methods

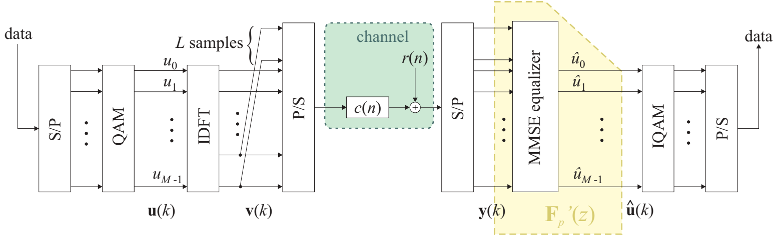
- **Windowing/Pulse Shaping**
  - shape the rectangular-windowed DMT symbols at the edges to improve selectivity
  - with some extra TD redundancy orthogonality can be kept
  - also helps for RFI cancellation and echo suppression at the US/DS band edges
  - standardized for VDSLx
- **Alternative Transform Bases**
  - overlapping basis filters → pulse shaping
  - example: Cos-Modulated Filterbanks (CMFB) → DWMT, "wDSL"



  - superior ISI/ICI robustness, even without cyclic prefix

Other Methods (cont'd)

- **MIMO Equalizer**



  - insufficient or no cyclic prefix, combine inverse transform and equalizer into large rectangular matrix
  - no sparse structure when combined with DMT transmitter

- **Fractional MIMO Equalizer**
  - instead of symbol-wise FFT, apply sliding FFT to the RX signal
  - FFT outputs at sampling rate, followed by MIMO equalizer → special case: PTEQ
  - similar to bank of N parallel filters → SIMO structure

Other Methods (cont'd)

- **DFE MIMO Equalizer**
  - instead of cyclic prefix, use Decision-Feedback Equalizer (DFE) for full equalization
  - difficult to apply for DMT, since decoding after the FFT
  - efficient structures proposed by Al-Dhahir and Cioffi (1995-1997)
- **Redundant Filterbanks**
  - general zero-forcing filterbanks with introduced redudancy
  - channel impulse response must be shorter than $P = M + L$
  - complexity
- **Transmitter Pre-Coding**
  - pre-distort transmit symbol according to channel characteristics to simplify equalization at the receiver side
  - Tomlinson-Harashima Pre-coding (THP) adopted for DMT with insufficient cyclic prefix by Cheong and Cioffi (1998)
  - increased transmit power, increased Crestfactor

Conclusions and Outlook

- DFT as the transform base for DMT/OFDM was not the optimal choice in terms of spectral containment of the subchannels → Cyclic Prefix was introduced
- further extensions to the original scheme (TEQ, Windowing, ...) necessary to improve spectral selectivity of the subchannels and thus to reduce ISI/ICI
- extended FEQ methods like GDMT and PTEQ are able to also incorporate other tasks like RFI suppression, Echo cancellation, Windowing, Crestfactor reduction, ...

⇒ joint optimization to reduce the amount of required redundancy