The McKay correspondence for quotient surface singularities

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The original McKay correspondence establishes a bijection between the set of conjugacy classes of finite subgroups $\Gamma \subset \mathrm{SL}\,(2, \mathbb{C})$ (that is - by abuse of language - the set of *binary polyhedral groups*) and the set of *Dynkin–diagrams* of type ADE via representation theory of Γ. This correspondence can be interpreted geometrically by

i) associating to a (finite dimensional complex) representation of Γ geometric objects on the *Klein singularity* $X_\Gamma := \mathbb{C}^2 / \Gamma$ resp. on its minimal resolution $\widetilde{X}_\Gamma$, and

ii) studying their intersection behaviour with the irreducible components of the *exceptional divisor* E_Γ.

For general *quotient surface singularities* X_Γ, Γ a finite (small) subgroup of $\mathrm{GL}\,(2, \mathbb{C})$, the situation is more complicated since there exist always fewer irreducible components of E_Γ than nontrivial irreducible representations. Nevertheless, one has a similar McKay correspondence if one considers only the smaller class of *special* representations.

I will discuss these older results together with a recent construction of $\widetilde{X}_\Gamma$ in terms of a Γ–invariant *Hilbert scheme* which gives a new understanding of the classical and the generalized geometric McKay correspondence.