The McKay correspondence for quotient surface singularities
Oswald Riemenschneider, Universität Hamburg

The original McKay correspondence establishes a bijection between the set of conjugacy classes of finite subgroups $\Gamma \subset \text{SL}(2, \mathbb{C})$ (that is - by abuse of language - the set of binary polyhedral groups) and the set of Dynkin-diagrams of type ADE via representation theory of $\Gamma$. This correspondence can be interpreted geometrically by

i) associating to a (finite dimensional complex) representation of $\Gamma$ geometric objects on the Klein singularity $X_\Gamma := \mathbb{C}^2/\Gamma$ resp. on its minimal resolution $\tilde{X}_\Gamma$, and

ii) studying their intersection behaviour with the irreducible components of the exceptional divisor $E_\Gamma$.

For general quotient surface singularities $X_\Gamma$, $\Gamma$ a finite (small) subgroup of $\text{GL}(2, \mathbb{C})$, the situation is more complicated since there exist always fewer irreducible components of $E_\Gamma$ than nontrivial irreducible representations. Nevertheless, one has a similar McKay correspondence if one considers only the smaller class of special representations.

I will discuss these older results together with a recent construction of $\tilde{X}_\Gamma$ in terms of a $\Gamma$–invariant Hilbert scheme which gives a new understanding of the classical and the generalized geometric McKay correspondence.