A quantum mechanical characterization of symmetric spaces

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In an influential paper published in 1962 ([D]) Freeman Dyson formulated and proved a universal law in a quantum mechanical setting which links that setting to a certain very restricted series of symmetric spaces. More recently (1995-2003) M. Zirnbauer and others observed that numerous real world quantum mechanical examples, e.g., involving superconductivity, are not covered by Dyson's model; in particular other symmetric spaces arise. In subsequent work (together with Zirnbauer and P. Heinzner [HHZ]) we give a final physical formulation of the quantum mechanical model and prove the analog of Dyson's Theorem.

Our basic result can be stated as follows: To every quantum system in the physics model there is a canonically associated classical compact symmetric space on the mathematics side. Conversely, every such symmetric space defines a quantum system. This is a mathematics theorem which is based on understanding the invariant theory of the situation.

In the talk both the physics and mathematics will be explained in elementary terms, the former from a mathematical standpoint. The above result will be stated in detail, and some indication of the methods of proof will be given.

References

[D] Dyson, F.J.: The threefold way: algebraic structure of symmetry groups and ensembles in quantum mechanics, J. Math. Phys. 3, 1199-1215 (1962)

[HHZ] P. Heinzner, A. Huckleberry and M. Zirnbauer: Symmetry classes of disordered fermions, (Preprint 52 pp., to appear in Comm. Math. Physics)