A Proof of Smale's Mean Value Conjecture

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In 1981 S. Smale conjectured:

Let $p \in \mathbb{C}\,[z]$ be a polynomial of degree $n > 1$ with $p(0) = 0$ and $p'(0) \neq 0$. Then

$$\min\{|p(\zeta)/(\zeta p'(0))| \,:\, p'(\zeta) = 0\} \leq 1\,.$$

Equality only occurs for $p(z) = a_1 z + a_n z^n$ with arbitrary $a_1, a_n \in \mathbb{C} \setminus \{0\}$.

The background of the conjecture was to estimate the complexity of algorithms approximating the zeros of polynomials. Smale proved the inequality with 4 instead of 1 on the right hand side, using classical results on conformal mappings (*The fundamental theorem of algebra and complexity theory*, Bull. Amer. Math. Soc. **4** (1981), 1–36). In fact the bound 1 can be sharpened a little bit by $\frac{n-1}{n}$.

In this talk a sketch of the proof will be given. The full version is available on the arXiv preprint server (`http://front.math.ucdavis.edu/math.CV`).