M-Day 2017

Jacobs University

October 13–14, 2017

Friday, October 13, 2017

13:15–13:30	*Welcome, IRC Seminar Room*
13:30–14:00	**Haidar Mohamad** (Jacobs University) *Optimal balance via adiabatic invariance of approximate slow manifolds*
14:00–14:30	**Richard Blender** (Universität Hamburg) *Fluctuation analysis of the atmospheric Lorenz energy cycle*
14:30–14:45	*Discussion*
14:45–15:30	*Coffee Break*
15:30–16:00	**Wei Pan** (Imperial College) *Numerically modelling stochastic Lie transport in fluid dynamics*
16:00–16:30	**Sergiy Vasylkevych** (Universität Hamburg) *Fluid models associated with the geodesic flow* *of H^1 metric on diffeomorphism groups* *as geometric generalised Lagrangian mean theories*
16:30–16:45	*Discussion*
16:45–17:30	*Poster presentations*
17:30–18:00	**Stephan Juricke** (Jacobs University) *First steps to backscatter in the Finite Element Sea-Ice Ocean Model*
18:00–18:30	**Nicolas Scharmacher** (Universität Hamburg) *Diffuse interface approaches in atmosphere and ocean –* *modeling and numerical implementation*
18:30–18:45	*Discussion*
19:00	*Leave for restaurant – meet in front of Campus Center*
19:30	**Dinner**, Restaurant “Zur gläsernen Werft” Schulkenstr. 2, 28755 Bremen

Saturday, October 14, 2017

9:00–9:30	**Armin Iske** (Universität Hamburg) *On the construction of meshfree finite volume particle methods*
9:30–10:00	**Claus Goetz** (Universität Hamburg) *Reconstruction based discontinuous Galerkin methods: features and challenges*
10:00–10:30	**Sergey Danilov** (AWI and Jacobs University) *On grid modes and discretizations on triangular meshes*
10:30–10:45	*Discussion*
10:45–11:30	*Coffee Break*
11:30–12:00	**Almut Gassmann** (IAP Kühlungsborn) *Deviations from a nonlinear wind balance: local and zonal mean perspectives*
12:00–12:30	**Christian Franzke** (Universität Hamburg) *Systematic decomposition of the MJO and its Northern hemispheric extra-tropical response into Rossby and inertio-gravity components*
12:30–12:45	*Discussion, Closing*

Poster presentations

Almut Gassmann (IAP Kühlungsborn)
Discretization of generalized Coriolis and friction terms on the deformed hexagonal C-grid

Denny Gohlke (Universität Hamburg)
Entropy production in turbulence parameterisations

Federica Gugole (Universität Hamburg)
An energy consistent stochastic 2-layer QG model

Florian Noethen (Universität Hamburg)
Convergence of Ginelli's algorithm for covariant Lyapunov vectors

Sebastian Schubert (Universität Hamburg)
The large deviation law and Lyapunov exponents in PUMA

Bastian Sommerfeld (IAP Kühlungsborn)
Entropy production in climate simulations

Abstracts

Richard Blender (Universität Hamburg)

Talk: *Fluctuation analysis of the atmospheric Lorenz energy cycle*

The Fluctuation Theorem relates the probabilities of negative and positive entropy productions in non-equilibrium physical systems. We analyze the atmospheric Lorenz energy cycle for the zonal mean and the eddy parts of the available potential and the kinetic energies. The cycle includes the injected power, the dissipated energy and internal conversions. The simulations are performed with the atmospheric model PUMA (Portable University Model of the Atmosphere, University of Hamburg) which is a dynamical core based on the hydrostatic primitive equations with linear forcing and friction. To overcome the problems of the missing reference state and the sparsity of negative data we shift the currents to reference states which represent typical states: the mode (pdf maximum) and the location parameter of a fitted Generalized Extreme Value (GEV) distribution. The time averaged current anomalies satisfy a Fluctuation Relation for small anomalies with linear slopes depending on the reference state. Large deviation rate functions for the energy currents collapse to a single function.

Sergey Danilov (AWI and Jacobs University)

Talk: *On grid modes and discretizations on triangular meshes*

Grid modes come into existence when a variable is placed at locations with different geometric properties, which allows decoupling between neighboring degrees of freedom. Depending on governing operators, grid modes induce spurious numerical modes, such as for example, spurious inertia-gravity waves of triangular C-grid or spurious inertial oscillations of triangular cell-vertex discretization. The manifestation of a grid mode depends on the governing equations and placement of other primitive variables. The origin of grid modes is purely geometrical. On triangular meshes the cell or edge placement of variables creates grid modes. A simple analysis of several discretizations will be presented, showing how spurious modes are triggered. In the context of energetic consistency, the presence of grid modes implies that they should be dissipated. This may create unphysical channels for energy dissipation.

Christian Franzke (Universität Hamburg)

Talk: *Systematic decomposition of the MJO and its Northern hemispheric extra-tropical response into Rossby and inertio-gravity components*

In this presentation I will present a mathematical framework for the systematic decomposition of three dimensional flow fields into Rossby and Inertio-Gravity waves. This decomposition is based in a normal mode decomposition using Hough functions as basis functions. Hough functions allow for balanced wind and geopotential height fields. The method will be evaluated by analysing the Madden-Julian Oscillation (MJO) in reanalysis data.

Almut Gassmann (IAP Kühlungsborn)

Talk: *Deviations from a nonlinear wind balance: Local and zonal mean perspectives*

The talk introduces the active wind as the deviation from a general local wind balance, the inactive wind. The inactive wind is directed along intersection lines of Bernoulli function and potential temperature surfaces. In climatological steady state, the inactive mass flux cannot participate in net-mass fluxes, because the mean position of the mentioned intersection lines does not change. A conceptual proximity of the zonal-mean active wind to the residual wind as occurring in the transformed Eulerian mean equations suggests itself.

The zonal- and time-mean active wind is compared to the residual wind for the Held-Suarez test case. Similarities occur for the meridional components in the zone of Rossby wave breaking in the upper troposphere equatorward of the jet. The vertical components are similar, too. However, the vertical active wind is much stronger in the baroclinic zone. This is due to the missing vertical eddy flux of Ertel's potential vorticity (EPV) in the TEM equations. The largest differences are to be found in the boundary layer, where the active wind exhibits typical pattern of Ekman dynamics.

Instantaneous active wind vectors demonstrate mass-inflow for lows and mass-outflow for highs in the boundary layer. An active meridional wind is associated with a filamentation of EPV in the zone of Rossby wave breaking in about 300 hPa. Strong gradients of EPV act as a transport barrier.

Almut Gassmann (IAP Kühlungsborn)

Poster: *Discretization of generalized Coriolis and friction terms on the deformed hexagonal C-grid*

Two nonlinear terms are investigated on hexagonal C-grid meshes: the generalized Coriolis term and the momentum diffusion term.

For a regular mesh, the mentioned terms are interpreted as finite difference representations in a trivariate coordinate system. This approach highlights that the vorticity and the shear deformation consist each of three subterms which are discretized on rhombi. Therefore, the full vorticity and shear deformation information is attached to a vertex, and the vertex comprises three rhombi.

A nonlinear instability which is due to the non-cancelation of subterms of the full momentum advection term is minimized by an a priori approach. Unlike in previous literature, where the stencil of the kinetic energy is enlarged a posteriori, the PV usage in the nonlinear generalized Coriolis term is modified. An analytic expression for the remaining cancelation error is given. It is heuristically discussed and experimentally verified that this remaining error is negligible.

Physical constraints determine the shape of the stress tensor. These are invariance to solid body rotation and a resulting positive definite dissipation rate. Strain and shear deformations are analytically proportional to $\partial_i u_i - \partial_j u_j$ and $\partial_j u_i + \partial_i u_j$, respectively. In these expressions, the two coordinate line combinations (x_1, x_2) and (x_1, x_3) must be treated separately. Unfortunately, a discretization of the analytical form of the shear deformation is impossible on the hexagonal mesh. Linear dependency relations between the wind and gradient components must be invoked in order to achieve consistency of the diffusion in stress tensor form with the diffusion in Laplacian form for constant diffusion coefficients.

Claus Goetz (Universität Hamburg)

Talk: *Reconstruction based discontinuous Galerkin methods: Features and challenges*

Two of the most popular families of high order schemes for hyperbolic conservation laws are reconstruction-based finite volume schemes and discontinuous Galerkin (DG) methods. The PN-PM philosophy presented in Dumbser et al. (J. Comput Phys. 227 (2008)) provides a unified framework for the treatment of both approaches. In a PN-PM scheme, the solution is represented in a finite element space of piecewise polynomials of degree N (hence the PN in the name of the method) and at each time step before the time evolution is carried out, a high order reconstruction of piecewise polynomials of degree $M > N$ is computed (hence the PM). In this framework, the pure DG method can be viewed as a PN-PN scheme, while the case P0-PM corresponds to the high order finite volume schemes. For $N > 0$ and $M > N$ a family of hybrid schemes emerges.

The PN-PM schemes have been applied to complex flow problem with good success, but our understanding of their analytical properties has not reached a mature level yet. We discuss some theoretical aspects of PN-PM schemes. In particular, we are concerned with the stability of these schemes and we show analytically why these methods are, in general, not L2-diminishing. To this end, we extend the famous cell square entropy stability result of Jiang and Shu (Math. Comput., 62 (1994)) for DG methods to the PN-PM case and identify which part in the reconstruction step may cause the instability. With this insight we design a flux limiter that enforces a cell square entropy condition for PN-PM schemes in 1D.

Denny Gohlke (Universität Hamburg)

Poster: *Entropy production in turbulence parameterisations*

The physically consistent representation of turbulence subgrid-scale processes in forced dissipative systems like atmosphere and ocean requires the handling of statistical nonequilibrium fluctuations. The statistics of these fluctuations — as a fingerprint of the chaotic dynamics — provide useful insights into the dynamical response behaviour of a system (transport coefficients) and could be restricted by fluctuation relations. The idea of this project is the analysis and the incorporation of suchlike constraints on fluctuations to modify existing parameterisation schemes, focusing on a stochastic and counter-gradient parameterisation of momentum and heat fluxes which are related to energy dissipation and backscatter.

Federica Gugole (Universität Hamburg)

Poster: *An energy consistent stochastic 2-layer QG model*

Following the paper of Frank and Gottwald (2013), we derived a stochastic version for the 2-layer quasi-geostrophic (QG) equations based on its Hamiltonian formulation. The stochastic terms have been introduced in such a way that the total energy is conserved. A parameter epsilon depending on the different time scales have been introduced to separate the barotropic and baroclinic modes. Next step will be to perform a stochastic mode reduction on the system, i.e. the baroclinic mode will be eliminated and we will derive an effective model only for the barotropic mode. We aim to develop a suitable stochastic solver in such a fashion that the resulting numerical model will be energy conserving. In our presentation we will discuss the results.

Reference: J. E. Frank and G. A. Gottwald, Stochastic homogenization for an energy conserving multi-scale toy model of the atmosphere, Physica D, 254:46-56, 2013.

This is joint work with Christian Franzke.

Armin Iske (Universität Hamburg)

Talk: *On the construction of meshfree finite volume particle methods*

This talk discusses the utility of meshfree kernel techniques in adaptive finite volume particle methods (FVPM). To this end, we provide supporting arguments in favour of kernel-based reconstructions in the recovery step of FVPM, where our discussion addresses relevant computational aspects concerning numerical stability and accuracy, as well as more specific points concerning efficient implementation. Special emphasis is finally placed on more recent advances in the construction of adaptive FVPM, where WENO reconstructions by polyharmonic spline kernels are used in combination with ADER flux evaluations to obtain high order methods for hyperbolic problems.

Stephan Juricke (Jacobs University)

Talk: *First steps to backscatter in the Finite Element Sea-Ice Ocean Model*

I will give an introduction to the concept of backscatter in ocean models and it's importance for an improved representation of eddy kinetic energy in the eddy permitting resolution range (around 1/10–1/4 degree resolution). First results of the backscatter parametrization in the Finite Element Sea ice Ocean Model will be presented, in a periodic channel test case with boundaries in the North and South. The new implementations lead to strongly increased eddy kinetic energy that corresponds to simulations with higher resolution. Future perspectives and applications will be discussed.

Haidar Mohamad (Jacobs University)

Talk: *Optimal balance via adiabatic invariance of approximate slow manifolds*

We analyze the method of optimal balance which was introduced by Viúdez and Dritschel (J. Fluid Mech. 521, 2004, pp. 343-352) to provide balanced initializations for two-dimensional and three-dimensional geophysical flows, here in the simpler context of a finite dimensional Hamiltonian two-scale system with strong gyroscopic forces. It is well known that when the potential is analytic, such systems have an approximate slow manifold that is defined up to terms that are exponentially small with respect to the scale separation parameter. The method of optimal balance relies on the observation that the approximate slow manifold remains an adiabatic invariant under slow deformations of the nonlinear interactions. The method is formulated as a boundary value problem for a homotopic deformation of the system from a linear regime where the slow-fast splitting is known exactly, and the full nonlinear regime. We show that, providing the ramp function which defines the homotopy is of Gevrey class 2 and satisfies vanishing conditions to all orders at the temporal end points, the solution of the optimal balance boundary value problem yields a point on the approximate slow manifold that is exponentially close to the approximation to the slow manifold via exponential asymptotics, albeit with a smaller power of the small parameter in the exponent. In general, the order of accuracy of optimal balance is limited by the order of vanishing derivatives of the ramp function at the temporal end points. We also give a numerical demonstration of the efficacy of optimal balance, showing the dependence of accuracy on the ramp function.

This is joint work with G. Gottwald and M. Oliver.

Florian Noethen (Universität Hamburg)

Poster: *Convergence of Ginelli's algorithm for covariant Lyapunov vectors*

Covariant Lyapunov vectors (CLVs) detect directions of different asymptotic growth rates of small linear perturbations to solutions in a dynamical system. During the last few years, several algorithms to compute CLVs emerged and were applied in a broad range of applications including climate models. One of the most popular algorithms was developed by Ginelli. It relies on the asymptotic characterization of CLVs as given by the Multiplicative Ergodic Theorem.

We motivate and explain Ginelli's algorithm in an exemplary setting giving the necessary theoretical framework. The center result is a convergence theorem, which reflects the observed convergence speed from computations. We will briefly discuss the proof. Furthermore, we expect that the theory of CLVs advances our understanding of the interaction of multiple types of instabilities in geophysical systems based on a conjecture of Gallavotti.

Sebastian Schubert (Universität Hamburg)

Poster: *The large deviation law and Lyapunov exponents in PUMA*

We have established that Covariant Lyapunov vectors (CLVs) allow for a systematic understanding of the energetics of the tangent linear perturbations in terms of the well known Lorenz energy cycle (LEC) (Schubert & Lucarini 2015). This study investigated the highly filtered quasi geostrophic equations, hence we could not expect that the CLVs represent different types of instabilities as suggested by Gallavotti (2013). In order to understand how multi-instability (baroclinic instability, gravity waves, etc) is represented in the splitting described by the CLVs, we focus our attention here on the primitive equation model PUMA which is the dynamical core of the Planet Simulator (PLASIM). As a first step, we analyze the first 200 Lyapunov exponents and explore whether we can construct a robust large deviation law. Note that in Vannitsem (2016) it was found that the rate functions are identical for all types of finite time Lyapunov exponents (backward, forward and covariant). We find that for near-zero exponents the convergence speed is considerably slower even though the auto-correlation is for all exponents near 2 to 3 days. Hence the presence of large deviation laws which allows for assessing the predictability on very long time scales is here not related to the auto-correlation of the observable itself.

References: Schubert, S. and Lucarini, V. (2015), *Covariant Lyapunov vectors of a quasi-geostrophic baroclinic model: analysis of instabilities and feedbacks*, Q.J.R. Meteorol. Soc., 141: 3040–3055; Gallavotti G. (2013), "Nonequilibrium and Irreversibility," Springer-Verlag; S. Vannitsem and V. Lucarini (2016), *Statistical and dynamical properties of covariant Lyapunov vectors in a coupled atmosphere-ocean model: multiscale effects, geometric degeneracy, and error dynamics*, J. Phys. A 49.

Wei Pan (Imperial College)

Talk: *Numerically Modelling stochastic Lie transport in fluid dynamics*

There are two main types of uncertainty in the prediction of weather and its variability. The first is the transport uncertainty of where the ow takes the thermodynamics. The second is the uncertainty in what the thermodynamics does when it gets there. We address the first type of uncertainty in our models, due to what we call "stochastic Lie transport". The framework for introducing cylindrical stochastic noise is based on the well known Hamilton's variational principle. This was first developed in D. Holm's 2015 paper.

The result is the Eulerian representation of ideal fluid dynamics in the form of stochastic partial differential equations, or SPDEs for short, where the stochastic perturbation is in the form of Stratonovich stochastic integrals of vector fields that depend on the gradients of the solution. This previously unseen form of noise is what we call transport noise.

For application purposes, e.g. data assimilation, the spatial correlation structure (SCS) of the transport noise is crucial and must be specified apriori. This leads to many important theoretical and practical questions regarding the determination of SCS as it is not possible to directly observe such quantity in practise. Thus, determining SCS would introduce its own uncertainty, but not even trying to determine them will mean neglecting the effects of transport uncertainty on weather prediction. As this form of noise is new, we propose the SCS components are to be supplied as spatial correlation EOFs of the data. We describe our estimation methodology, and how we benchmark the validity of the estimation. We aim to answer important questions such as the interpretation of SCS, the choice on the number of SCS and the implications of these choices on the Eulerian distributional properties of the velocity and vorticity, and the impact on practical computations. This is the first step of a larger data assimilation project which we are embarking on.

This is joint work with C. Cotter, D. Crisan, D. Holm and I. Shevchenko.

Nicolas Scharmacher (Universität Hamburg)

Talk: *Diffuse interface approaches in atmosphere and ocean – modeling and numerical implementation*

TBA

Bastian Sommerfeld (IAP Kühlungsborn)

Poster: *Entropy production in climate simulations*

The physically consistent representation of turbulence subgrid-scale processes in forced dissipative systems like atmosphere and ocean requires the handling of statistical nonequilibrium fluctuations. The statistics of these fluctuations — as a fingerprint of the chaotic dynamics — provide useful insights into the dynamical response behaviour of a system (transport coefficients) and can be restricted by fluctuation relations. The idea of this project is the analysis and the incorporation of suchlike constraints on fluctuations to modify existing parameterisation schemes, focusing on a stochastic and counter-gradient parameterisation of momentum and heat fluxes which are related to energy dissipation and backscatter.

Sergiy Vasylkevych (Universität Hamburg)

Talk: *Fluid models associated with the geodesic flow of H^1 metric on diffeomorphism groups as geometric generalised Lagrangian mean theories*

Many phenomena in fluids can be effectively analysed in terms of interaction between the mean flow and fluctuations. Lagrangian averaging expresses fluctuations in terms of particle displacement, which results in equations that preserve the geometric structure of the original problem. Geometric generalized Lagrangian mean is the Lagrangian averaging method recently proposed by Gilbert and Vanneste that extends GLM theory of Andrews and McIntire from Euclidean space to the manifold setting.

It is well known that Euler's equation of ideal fluid arise as the geodesic flow on volume preserving diffeomorphism group equipped with L^2 metric. Likewise, the geodesic flow on the full diffeomorphism group is given by Burgers' equations. We show that application of GGLM to L_2 geodesic motion on diffeomorphism groups yields H^1 geodesic flow on the same group. This derives LAE (Euler-α), Camassa-Holm, and EPDIFF equations as a GGLM theory for Euler and Burgers' equations in one and multiple dimensions, respectively.