Engineering and Science Mathematics I
Standard Track

Midterm I

October 9, 2002

1. Compute the following limits.

   (a) $\lim_{z \to 3} \frac{2 - \sqrt{1+z}}{3 - z}$

   (b) $\lim_{r \to -1} \frac{r^2 - 1}{2r^2 + 3r + 1}$

   (c) $\lim_{w \to 0} \frac{w}{\sin \sqrt{w}}$

(5+5+5)

2. The following functions are discontinuous at a point. Can you remove the discontinuity? Explain.

   (a) $g(y) = y^2 \cos \frac{1}{y}$

   (b) $h(\theta) = \frac{\theta}{|\theta|}$

   (c) $\psi(r) = \frac{r}{1 - \cos r}$

(5+5+5)

3. Differentiate the following functions.

   (a) $\ell(t) = \frac{(\sin t)^2}{\cos t}$

   (b) $m(p) = e^{-\ln p}$

   (c) $n(q) = \frac{\sin(q^2)}{\cos q}$

(5+5+5)

4. Find all points (x, y) on the graph of $x^{2/3} + y^{2/3} = 8$ where lines tangent to the graph have slope 1. (10)

5. Use the definition of the derivative to show that $\cos' \theta = -\sin \theta$. (10)

6. Find the global minimum and maximum values, if they exist, of the function

$$f(x) = \frac{x^2 + 1}{(x - 1)^2}$$

(10)

7. A square sheet of cardboard of width W will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume? (10)

8. A farmer owns a 10 km long stretch of land between two parallel rivers that are 2 km apart. What is the area of the largest rectangular corral he can enclose with

   (a) 2 km of fencing,

   (b) 5 km of fencing,

   assuming that no fence is needed along the river. (10+5)