

Engineering and Science Mathematics I

Standard Track

Midterm II, Version A

November 11, 2002

1. Consider the function

$$f(x) = \frac{\ln x}{x^3}.$$

Find the domain, intercepts, horizontal and vertical asymptotes, extrema and inflection points of f . Identify all minima and maxima of f , as well regions where the graph is concave upward or concave downward. Finally sketch the graph into the coordinate system provided. (20)

2. Compute the following indefinite integrals.

(a) $\int \frac{1}{(1+x)\sqrt{x}} dx$

(b) $\int \frac{x^4}{x^2-2} dx$

(c) $\int \frac{\tan x}{\ln \cos x} dx$

(d) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

(e) $\int \sin \sqrt{x} dx$

(6+6+6+6+6)

3. Compute the following definite integrals.

(a) $\int_{-178}^{178} \sin x \ln\left(\frac{2+\cos x}{2-\cos x}\right) dx$

(b) $\int_1^e x^4 (\ln x)^3 dx$

(6+6)

4. Evaluate the following improper integrals. Careful: some of the integrals may not converge.

(a) $\int_0^2 \frac{1}{1-x} dx$

(b) $\int_1^2 \frac{1}{\sqrt{x-1}} dx$

(6+6)

5. Find all values for b for which the integral

$$\int_{-\infty}^{\infty} b r e^{br^2} dr$$

converges. What is the value of the integral when it converges? (6)

6. Find the volume of the ellipsoid that is obtained by revolving the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about the y -axis. (10)