Partial Differential Equations

Homework 3

due October 7, 2002

1. Prove that the Taylor series with integral remainder for a function $u \in C^N(\mathbb{R}^n)$ is
$$u(x) = \sum_{|\alpha| \le N-1} \frac{D^\alpha u(x_0)}{\alpha!} (x - x_0)^\alpha + R_N,$$
where
$$R_N = N \sum_{|\alpha|=N} \frac{(x-x_0)^\alpha}{\alpha!} \int_0^1 (1-t)^{N-1} D^\alpha u(x_0 + t(x-x_0))\, dt.$$
Hint: Apply the one-dimensional Taylor formula with integral remainder to the function $f(s) = u(x_0 + s(x - x_0))$.

2. Evans, p. 86/87 problems 5 and 6, 8.