Numerical Methods I

Review for Midterm Exam

Friday, October 17, 2003

1. Computer Arithmetic: Understand how floating point numbers work (you do not need to know implementation details, but should have an understanding of effects as exemplified in Homework 1 Questions 2, 5).

2. Condition number: definition (for the evaluation of a real-valued function, for finding roots, and also for matrices), what does it tell you? (See Homework 1 Questions 3, 4; Homework 3 Questions 2b, 5.)

3. Root finding: Bisection method, Newton method; have an idea how the secant method can be derived from the Newton method.

4. Order of convergence: hands-on definition of order of convergence; how can you motivate by Taylor expansion that Newton's method is (usually) convergent of order 2? When does Newton's method fail to converge quadratically?

5. Contraction mapping theorem: Understand basic idea. (Think about connection to iterative solvers for linear systems!)

6. Matrix norms: Definition, connection with condition numbers (see Homework 3, Questions 1, 2).

7. LU and QR factorizations of a matrix; how to use the factored form to solve a system of linear equations; importance of pivoting. (No need to study implementation details, but be familiar with matrix manipulations, especially multiplication of block matrices, and the basic ideas behind the algorithms.)

8. Can you compute by hand the LU and QR decompositions of simple matrices (2×2 or at most 3×3)?

9. Least squares: How to use QR factorization to solve least squares problems.

10. Iterative solvers for linear systems: Jacobi and Gauss–Seidel method, convergence criterion for general iterative methods, basic idea behind gradient and conjugate gradient method.