Numerical Methods I – Problem Set 2

Homework due in class, September 19, 2003

Projects due September 22, 2003

1. Let $g\colon [a,b] \to \mathbb{R}$ be a contraction, i.e.

$$|g(x) - g(y)| \leq L\,|x - y|$$

for some $0 < L < 1$ and for all $x, y \in [a,b]$. Let ξ denote the fixed point of g. Give an estimate on how many iterations you need to guarantee, for given $\varepsilon > 0$, that

$$|g(x) - \xi| \leq \varepsilon\,.$$

2. As a rule of thumb, quadratic convergence doubles the number of accurate digits each iteration. However, this is not always true. Give an estimate on the number of accurate digits you gain each Newton iteration in terms of f' and f''.

3. Work out either (a) or (b).

   (a) Prove that the secant method converges with order $q = \frac{1}{2}(1+\sqrt{5})$. Detailed hints are available from SM, Exercise 1.10.

   (b) Estimate, by performing an appropriate log-log plot in Octave, the rate of convergence of the secant method.

4. **Project:** Write three Octave routines to find a zero of a function $f(x)$ by using

   (a) Newton's method with starting value x_0,

   (b) the secant method with two starting values x_0 and x_1,

   (c) the bisection method on an initial interval bounded by a and b.

5. **Project:** (Example from QS.) Test your root finding routines on each of the functions

   (a) $f(x) = \cosh x + \cos x - 3$

   (b) $g(x) = \cosh x + \cos x - 2$

   and compare convergence and accuracy.