General Mathematics and Computational Science I

Exercise 6

September 22, 2005

1. (Bernoulli inequality.) Use induction to prove that for every $x \in \mathbb{Q}$ with $x \geq -1$ and every $n \in \mathbb{N}$,
$$(1+x)^n \geq 1 + n\,x\,.$$

2. Recall the definition of addition and multiplication on the rationals. For $x = [a/b] \in \mathbb{Q}$ and $y = [c/d] \in \mathbb{Q}$, we define $x + y = [(ad + bc)/bd]$ and $x \cdot y = [ac/bd]$.

   Show that multiplication is well-defined. I.e., show that the definition is independent of the choice of representative under the equivalence relation $(a, b) \sim (a', b')$ if $a \cdot b' = a' \cdot b$.

3. We say a field $\mathbb{F}$ is *ordered* if there exists a relation $>$ which respects addition and multiplication in the following way. For any $x, y, z \in \mathbb{F}$,

   (O1) if $x > y$, then $x + z > y + z$;

   (O2) if $x > y$ and $z > 0$, then $xz > yz$.

   Recall from class that the set of rational numbers $\mathbb{Q}$ can be ordered as follows. A rational number, i.e. an equivalence class $[a/b] \in \mathbb{Q}$, is *positive* if $ab > 0$ and *negative* if $ab < 0$. Then, for rationals $x = [a/b]$ and $y = [c/d]$, we define $x > y$ if $x - y > 0$.

   Show that this order relation satisfies (O1) and (O2).