

# General Mathematics and Computational Science I

## Exercise 10

October 13, 2005

1. Show that

$$\binom{n+1}{k+1} = \sum_{j=k}^n \binom{j}{k}$$

for every  $0 \leq k \leq n$ .

2. Use the method of generating functions to find a closed form expression for the members of the generalized Fibonacci sequence

$$\begin{aligned}a_0 &= A, \\a_1 &= B, \\a_n &= a_{n-1} + a_{n-2}.\end{aligned}$$

3. Use the method of generating functions to find a closed form expression for the members of the sequence

$$\begin{aligned}b_0 &= 1, \\b_1 &= \cos \theta, \\b_n &= 2 \cos \theta b_{n-1} - b_{n-2},\end{aligned}$$

where  $\theta$  is an arbitrary real number.