

# General Mathematics and Computational Science I

## Exercise 11

October 18, 2005

1. Recall that the generalized binomial coefficients are defined as the coefficients in the formal power series

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k.$$

Show that

$$\binom{\alpha}{k+1} = \frac{\alpha - k}{k+1} \binom{\alpha}{k}$$

for every real number  $\alpha$ , and  $0 \leq k$ .

2. Use the method of generating functions to find a closed form expression for the members of the sequence

$$c_0 = 1,$$
$$c_{n+1} = \sum_{k=0}^n c_k c_{n-k}.$$

*Hint:* Your answer will involve generalized binomial coefficients with  $\alpha = \frac{1}{2}$ , see Question 1 above. You may leave your answer in terms of these coefficients; there is no need to further expand the expressions although you may want to check the first couple of terms to see whether your answer is correct.