

# General Mathematics and Computational Science I

## Exercise 16

November 8, 2005

1. Check the conditions for Laplace's method for the function

$$f(x) = x - \ln x - 1$$

which appears in the proof of Stirling's formula. I.e., check the following.

- (a)  $f$  is strictly decreasing for  $0 < x < 1$ , strictly increasing for  $x > 1$ , and  $f(1) = 0$ .
- (b) There are positive constants  $b$  and  $c$  such that  $f(x) \geq bx$  for  $x \geq c$ .
- (c)  $f(x) = a(x-1)^2 + \psi(x)(x-1)^3$  where  $\psi$  is a bounded function for  $x \in [1-\delta, 1+\delta]$  for some  $\delta > 0$ . How must  $a$  be chosen?

Hint: For (c), use either Taylor's formula with remainder (easy), or L'hôpital's rule for the function

$$g(x) = \frac{f(x)}{(x-1)^2}.$$

(more elementary, but longer).

2. (From Lial *et al.*) Solve the following linear programming problem: Maximize

$$z = 4x + 2y$$

subject to

$$\begin{aligned}x + y &\leq 5, \\2x + y &\geq 2, \\x + 2y &\geq 3, \\x &\geq 0, \\y &\geq 0.\end{aligned}$$

3. (From Lial *et al.*) A farming community owns 2 acres of land and can afford to devote 40 days a year to harvesting. Two crops can be grown in the region, millet and wheat, which produce 200 kg and 400 kg per acre, respectively. Millet requires 10 days per acre to harvest, while wheat requires 30 days per acre. How many acres should be devoted to each crop to maximize the weight of grain harvested?