Numerical Methods I – Lab 2

Fall Semester 2005

September 20, 2005

1. Determine, numerically, the error behavior of Problem 1 from Lab 1.

   Recall: The limit

   $$C = \lim_{n\to\infty} c_n = 0.577\,215\,664\,901\,532\ldots \qquad \text{with} \qquad c_n = \sum_{k=1}^{n} \frac{1}{k} - \ln n$$

   is called Euler constant.

   (a) Write an Octave function for computing c_n which accepts as optional input c_m for $1 \le m < n$. Test its performance (runtime and absolute error with respect to the given value for C) for $n \le 10^7$.

   (b) Verify numerically (again for $n \le 10^7$) that the approximation error satisfies

   $$E_n \equiv c_n - C = \frac{1}{2n} + O\Big(\frac{1}{n^2}\Big)$$

   as $n \to \infty$. Assuming that this relation holds, compute a new sequence

   $$d_n = 2\,c_{2n} - c_n$$

   and check the new error sequence $F_n \equiv d_n - C$ numerically. Explain the result!

2. Show, numerically, that the iteration

   $$y_{n+1} = 1 - \cos(y_n)$$

   converges to zero with order 2.