General Mathematics and Computational Science I

Exercise 9

October 24, 2006

1. Recall that the generalized binomial coefficients are defined as the coefficients in the formal power series
$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \,.$$
Show that
$$\binom{\alpha}{k+1} = \frac{\alpha-k}{k+1}\binom{\alpha}{k}$$
for every real number α, and $0 \le k$.

2. Use the method of generating functions to find a closed form expression for the members of the sequence
$$c_0 = 1\,,$$
$$c_{n+1} = \sum_{k=0}^{n} c_k \, c_{n-k} \,.$$
Hint: Your answer will involve generalized binomial coefficients with $\alpha = \frac{1}{2}$, see Question 1 above. You may leave your answer in terms of these coefficients; there is no need to further expand the expressions although you may want to check the first couple of terms to see whether your answer is correct.