General Mathematics and Computational Science I

Exercise 17

November 28, 2006

1. On the previous homework, you have shown that the logistic map

$$x_{n+1} = \mu\, x_n\, (1 - x_n)$$

   has a 2-cycle whenever $\mu > 3$ with

   $$a = \frac{1 + \mu - \sqrt{(\mu - 3)(\mu + 1)}}{2\,\mu}\,,$$
   $$b = \frac{1 + \mu + \sqrt{(\mu - 3)(\mu + 1)}}{2\,\mu}\,.$$

   Use the result from Exercise 15, Question 3 to prove that this 2-cycle is asymptotically stable for $3 < \mu < 1 + \sqrt{6}$ and unstable for $\mu > 1 + \sqrt{6}$.

2. Solve the so-called *Pielou logistic equation*

   $$x_{n+1} = \frac{\alpha\, x_n}{1 + \beta\, x_n}\,,$$

   find its equilibrium points and determine their stability.

   Hint: To solve the equation, substitute $x_n = 1/y_n$.