# General Mathematics and Computational Science I

Midterm II

November 7, 2006

1. In chemistry, benzene is a molecule made of six carbon atoms forming a hexagon, each of which also bonds to a hydrogen atom (H). In how many ways can hydrogen atoms be substituted by two methyl groups (M) and one ethyl group (E) as to form distinct molecules?

   Three possible configurations are shown as examples:
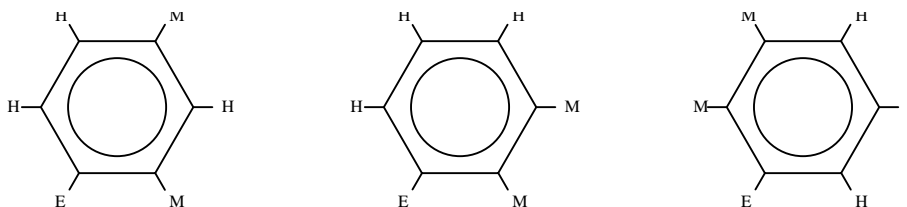
   Note that the left two examples are distinct molecules, while the right two are a pair of identical molecules!

2. Prove the so-called *Bernoulli inequality*

$$(1 + x)^n \geq 1 + n\,x$$

   for any real number $x > -1$ and $n \in \mathbb{N}$. (8)

3. (a) You flip a coin five times in a row. What is the probability that it comes up heads five times?

   (b) You flip a coin ten times in a row. Which of the following outcomes is more likely? HHHHHHHHHH or THTTHHTHHT? Explain!

   (6+4)

4. Use the method of generating functions to find a closed form expression for the members of the sequence
$$a_{n+1} = 2\,a_{n-1} - a_n$$
   where $a_0 = 1$ and $a_1 = -2$. (8)

5. Find the largest value of the function $f(x, y) = xy$ on the set of points satisfying the inequality $x^2 + xy + y^2 \leq 1$. (8)

6. Show that
$$(a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq 9$$
for arbitrary positive real numbers a, b and c. When does equality hold? (8)