Engineering and Science Mathematics 2B

Homework 6

due March 22, 2003, before 22:00

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. Let f be a function on the interval $[0, 2\pi]$ with Fourier representation

$$f(x) = \frac{1}{\sqrt{2\pi}} \sum_{k=-\infty}^{\infty} c_k \, e^{ikx} \, . \tag{*}$$

   Show that if f is real, then $c_k^* = c_{-k}$.

2. Assume f is as in (*). Find the Fourier coefficients for

   (a) $f(x - x_0)$ where x_0 is a constant,

   (b) $f(-x)$,

   (c) $f^*(x)$,

   (d) $\int_s^t f(\xi) \, d\xi$, assuming that $c_0 = 0$,

   (e) $f'(x)$.

3. Compute the Fourier cosine series of $f(x) = |x|$ on the interval $[-\pi, \pi]$.

4. (E) Let

$$\boldsymbol{v} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix} .$$

   Compute the projection of $\boldsymbol{v}$ onto the subspace spanned by the orthonormal vectors

$$\boldsymbol{e}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} , \quad \boldsymbol{e}_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} .$$

(A) An odd function of period 2π is approximated by a Fourier sine series having only N terms. The error in the approximation is measured by the square deviation

$$E_N = \int_{-\pi}^{\pi} \left[f(x) - \sum_{n=1}^{N} b_n \frac{\sin nx}{\sqrt{\pi}} \right]^2 dx \,.$$

By differentiating E_N with respect to the coefficients b_n, find the values of b_n that minimize E_n.

5. Give the value to which the Fourier series of the function $f(x) = (x + \pi)^2$, defined on the interval $[-\pi, \pi]$, converges at each of the following points: $x = -\pi, 0, \pi, 2\pi$.

6. Compute the complex Fourier series of the function $f(x) = e^x$ on the interval $[-\pi, \pi]$.