

Engineering and Science Mathematics 2B

Homework 10 – Quiz Only!

The last day to take the quiz is Wednesday, May 19, 2004

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. In a promotional quiz each player, independently, has the chance of winning 1000 Euros with probability p . It is obviously bad for the organizer if more than one person wins, but it is also considered bad for the promotion if nobody wins at all.
 - (a) With 500 players participating, what is the optimal choice for p , i.e. the one that maximizes the chance of exactly one player winning?
 - (b) For this value of p , what is the expected payout?
2. Let X be the number of emails you receive in any given hour, and assume that X is Poisson distributed with parameter $\lambda = 10 \text{ h}^{-1}$.
 - (a) What are the probabilities of receiving (i) no, (ii) exactly six, (iii) at least eight email messages in this hour?
 - (b) Use the approximation of the Poisson by a normal distribution to re-answer the questions from part (a).
3. Let X_1, X_2, \dots be independent discrete random variables taking values in $\{-\frac{1}{2}, \frac{1}{2}\}$ with probability function

$$p(-\frac{1}{2}) = q, \quad p(\frac{1}{2}) = (1 - q),$$

and $p(x) = 0$ for all other values of x .

(E) Find the expectation for the random variable

$$Z_N = \frac{X_1 + \dots + X_N}{N}$$

as $N \rightarrow \infty$.

(A) Let $Y_n = 2^{-n} X_n$. Find the expectation for the random variable

$$Z_N = Y_1 + \dots + Y_N$$

as $N \rightarrow \infty$.