

Engineering and Science Mathematics 2B

Homework 2

due February 18, 2004

Normal questions and advanced questions (A) are worth 5 points; easy questions (E) are worth 4 points. Complete either the easy, or the advanced version, not both.

1. A matrix A is called normal if $AA^H = A^H A$, where A^H denotes the Hermitian conjugate of A . Show (a) that $(A^H)^{-1} = (A^{-1})^H$, and (b) that A^{-1} is normal if A is normal.

2. Let

$$B = \begin{pmatrix} 0 & -i & i \\ i & 0 & -i \\ -i & i & 0 \end{pmatrix}, \quad C = \frac{1}{\sqrt{8}} \begin{pmatrix} \sqrt{3} & -\sqrt{2} & -\sqrt{3} \\ 1 & \sqrt{6} & -1 \\ 2 & 0 & 2 \end{pmatrix}.$$

Are these matrices real, diagonal, symmetric, antisymmetric, singular, orthogonal, Hermitian, anti-Hermitian, unitary, and/or normal?

3. (E) Let

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 3 & 4 \end{pmatrix}.$$

Compute AB and BA . Conclude that (i) $AB \neq BA$, and (ii) that $AB = 0$ does not imply that either A or B is the zero matrix.

- (A) Prove that $AB = 0$ implies that at least one of the matrices is singular.

4. (E) Determine if the following vectors are linearly dependent or linearly independent:

$$\begin{pmatrix} -4 \\ 0 \\ 1 \\ 5 \end{pmatrix}, \quad \begin{pmatrix} -3 \\ -1 \\ 0 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 4 \\ 3 \\ 6 \end{pmatrix}.$$

- (A) Prove the following statement: Let $\mathbf{v}_1, \dots, \mathbf{v}_n$ be linearly independent. If a vector \mathbf{w} can be written

$$\mathbf{w} = \sum_{k=1}^n \alpha_k \mathbf{v}_k,$$

the choice of the coefficients $\alpha_1, \dots, \alpha_n$ is unique.

5. Solve the following system of linear equations using the method taught in class.

$$\begin{aligned}x_1 + 3x_2 - 5x_3 &= 4 \\x_1 + 4x_2 - 8x_3 &= 7 \\-3x_1 - 7x_2 + 9x_3 &= -6\end{aligned}$$

6. (E) Solve the following system of linear equations using the method taught in class.

$$\begin{aligned}x_1 + 3x_2 + x_3 &= 1 \\-4x_1 - 9x_2 + 2x_3 &= -1 \\-3x_2 - 6x_3 &= -3\end{aligned}$$

(A) Find conditions on α such that following system of linear equations has (a) exactly one solution, (b) no solutions, or (c) an infinite number of solutions; give all solutions where they exist.

$$\begin{aligned}x_1 + \alpha x_2 &= 1 \\x_1 - x_2 + 3x_3 &= -1 \\2x_1 - 2x_2 + \alpha x_3 &= -2\end{aligned}$$