

Numerical Methods II

Problem Set 8

due in class, May 12, 2004

1. **Project:** Write an Octave program to verify experimentally that

$$\mathbb{E} \left| \sum_{i=0}^{n-1} (W_{i+1} - W_i)^2 - T \right|^2 \rightarrow 0 \quad \text{as } n \rightarrow \infty,$$

where $W(t)$ is Brownian motion, $t_i = i \delta t$ with $\delta t = T/n$, and $W_i = W(t_i)$.

2. Use the Itô formula to show that the so-called *Itô equation*

$$dX = dt + 2\sqrt{X} dW$$

has the explicit solution

$$X(t) = (W(t) + \sqrt{X(0)})^2.$$

3. **Project:** Write an Octave program to solve the Itô equation with deterministic initial data $X(0) = 1$ with Milstein's method. Show that the method converges with strong order 1.

Hint: For both programming projects, you can use the programs from Desmond Higham's article with minor modifications.