10 Ethnomusicology, Ethnomathematics. The Logic Underlying Orally Transmitted Artistic Practices

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Ethnomathematics is a new domain that has arisen during the last two decades, at the crossroad between history of mathematics and mathematics education. This domain consists in the study of mathematical ideas shared by orally transmitted cultures. Such ideas are related to number, logic and spatial configurations [9,11]. My purpose is to show how ethnomusicology could turn musical materials in this direction. Music will be considered here as a mean of organizing time through patterns of sound events. Thus we shall focus on musical forms and structures, rather than on other aspects of music (such as social aspects for instance). We will ask whether particular forms of traditional music share specific properties, namely combinatorial properties, that could be of some interest from an ethnomathematical point of view.

The study of mathematical ideas of non-literate peoples goes against persisting notions in the mathematics literature, which are strongly influenced by the late nineteenth-century theory of classical evolution. According to this theory, cultures can be ordered on an intellectual scale from primitive peoples to Western culture [43]. These ideas have been quite influential in mathematics literature and continue to be cited [30]. Another idea developed by [33,34] introduced a distinction between the Western mode of thought, and a "prelogical" mode of thought (or "unscientific" as Evans-Pritchard said) characterizing traditional peoples. A rich debate has arisen from this controversial theory, involving anthropologists, cognitive psychologists, and philosophers, on the nature of "rationality", with special attention paid to the witchcraft problem of the Zande peoples from Sudan [23,46,28,29,26], with echoes in [27,31] and others. Ethnomathematics has grown in the wake of this epistemological debate, gathering mathematicians from different parts of the world including the southern hemisphere. A research program has been sketched, and an International Study Group was founded [2].

The efforts made by ethnomathematicians in order to correct erroneous theories on the ability of human thought to think abstractly or logically rely greatly on the works of former ethnologists who have recorded information involving mathematical ideas while doing field work at the end of the nineteenth or during the twentieth century. Not being especially engaged with mathematics in their own culture, these ethnologists did not extract the whole mathematical content of their recorded material. Thus a great amount of work remains in the study of this field material from a mathematical point of view. In the case of music, recorded material will consist in various forms of written transcriptions of the music, as we shall see in the examples discussed in this paper.

Talking about "mathematics" in the context of orally transmitted societies requires some preliminary remarks, since mathematics are sometimes considered a Western invention [42]. There is an implicit assumption underlying this approach asserting that the practices we are studying share something in common with Western mathematics. In fact, both are linked through phenomena we have called "mathematical ideas". The concept of number, for instance, is a mathematical idea which seems to be universal [18,47]. But the scope of mathematical ideas is not clearly delineated. Furthermore, the way these ideas are expressed and their context in human thought vary from culture to culture. As Daniel Andler pointed out during his lecture at the Diderot Forum, there may exist a gap between the formal properties of traditional objects (such as geometric drawings, for instance), which are discovered by ethnomathematicians and expressed in their own mathematical language, and the cognitive processes of peoples who produced these objects. This is particularly true since the studies are generally based on recorded materials collected during field works made in the past, without interacting with the native peoples. This leads to the question whether the formal properties discovered in these field materials reflect a conscious activity of the mind. The answer to this question determines the cognitive level of our ethnomathematical descriptions. Even in Western mathematics, one can distinguish different cognitive levels, as pointed out in [21] following the ideas of [32]. One of these levels is the formalized text written by mathematicians in professional publications. But the mathematician's activity involves many other levels, including simple "reveries" in which mathematical ideas are put together in an involuntary way.

Ethnomathematical studies attempt to order mathematical practices of nonliterate peoples on the scale of different cognitive levels. Since the work of Piaget [36], new methods have been developed in psychology for cross-cultural studies [17], as applied in the analysis of the strategies of players of a well-known African game called "awele" [38]. The development of cognitive anthropology is the result of this growing interest for the cognitive aspects of ethnological studies [5,41]. In the examples discussed in this paper, we shall try to indicate to what extent the formal properties we are studying are explicit in the mind of the native peoples, but as we shall see, this is not always possible without interacting with them.

My presentation is divided into three parts. The first part deals with sand drawing from the Vanuatu, a non-musical example presented here in order to show what kind of ideas is studied in ethnomathematical writings. Decorative arts have been the first activity of nonliterate peoples which became the subject of mathematical investigations, from the point of view of their symmetry properties, involving the classification developed by crystallographers and adapted by [37] to the two dimensional case [40,45]. In the case of

sand drawings, the mathematical analysis involves formal languages, as it is described for the *kolam* of India in [39], but also graph theory. The beautiful Vanuatu figures we shall present in this paper have been studied by various mathematicians who have been interested in the properties of their tracing paths [10,11,24].

Following this preliminary example taken from visual art, I will present two musical examples, in order to show how musical practices could bring new insights in the development of ethnomathematics. The first musical example concerns harp music from Nzakara people of Central African Republic. I have been working on this repertory for ten years in collaboration with Éric de Dampierre [20]. As we shall see, the short harp formulas played by traditional musicians share interesting combinatorial properties.

The last example deals with polyrhythmic music from the Aka Pygmies, which has become famous since the works of ethnomusicologist Simha Arom [6]. He has discovered interesting properties of asymmetric rhythms underlying this music, and we shall focus on the combinatorics of these rhythmic patterns.

10.1 Sand Drawings from the Vanuatu

10.1.1 The Guardian of the Land of Dead

We first turn to a country where there is a rich tradition of tracing figures in the sand, the Republic of Vanuatu, called the New Hebrides before its independence in 1980. This chain of some eighty islands is located in the South Pacific, 200 kilometres northern-east of New Caledonia. On some of these islands (mainly Malekula, Ambrym and Pentecost), the tradition of tracing figures in the sand has produced many interesting and sophisticated figures.

The technique used to draw these figures simply consists in tracing on the sand with the finger. Often a framework of a few horizontal and vertical lines is drawn before tracing the figure itself. This practice is still in use, as one can see by looking at recent pictures of traditional sand drawings in the catalogue of the exhibition devoted to arts from the Vanuatu at the Musée des Arts d'Afrique et d'Océanie in Paris in 1997 [44], or in a little book by Jean-Pierre Cabane [13]. This part of my paper took originally the form of a concert-conference at the Musée, which was initiated by composer Tom Johnson who wrote a piece of music for contrabass saxophone following the tracing of the tortoise (which is reproduced in Fig. 10.3 below) [16].

The tracing paths of these figures satisfy a rule, which is strongly related to graph theory. Figures "are to be drawn with a single continuous line, the finger never stopping or being lifted from the ground, and no part covered twice" [11, p. 45]. Moreover when the drawing ends at the point from which it began, it is called *suon*. This rule for tracing figures corresponds to what is called *Eulerian path* in graph theory, that is a continuous path that covers each edge of a graph once and only once. Finding a Eulerian path in a given graph is not trivial, and sometimes not even possible in situations such as the famous seven bridges of Königsberg, as Euler proved it in 1736.

Euler's statement provides a good example for illustrating the different cognitive levels found in mathematical activity. The necessary condition of the statement relies on a simple idea: finding a Eulerian path in a graph requires that for each vertex the number of adjacent edges is even, except two of them, since a vertex being reached by one edge must be left by another. Thus adjacent edges can be grouped by pairs, except those emanating from the beginning and ending points. The sufficient condition is a more formal result, asserting that whenever all numbers of edges emanating from the same vertex are even, except for two vertices, then a Eulerian path can be found. The proof develops the previous simple idea in the form of a more technical induction argument on the number of vertices.

An important point concerning this tracing rule is that most of the drawings are related to myths, some of them emphasizing the mathematical property of the figures expressed by the tracing rule. Often tracing figures in the sand is achieved while somebody is telling a story associated with the figure. Thus myths appear to be comments of related figures. The tracing rule itself is sometimes part of the myth, as it is the case for a specific figure related to the passage to the Land of the Dead. In the myth, this figure is said to be traced on the sand by the guardian of the Land of the Dead. The guardian challenges those who try to enter. When the ghost of a newly dead person approaches, the guardian traces half the figure as shown in Fig. 10.1 [13, p. 18]. The ghost has then to complete the bottom part of the figure with a continuous path. If he fails, he is eaten by the guardian. As one can see, the challenge consists precisely in finding a Eulerian path in a graph, thus being strongly related to the mathematical property of the figure.

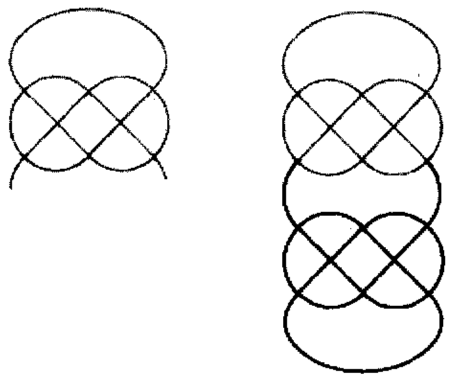

Fig. 10.1. Challenge to access the Land of the Dead

Other figures are associated with stories that refer directly to the tracing of the figure. One of them is called "Rat eats breadfruit, half remains": "First a figure described as a breadfruit is drawn completely and properly. Then the retracing of some edges is described as a rat eating through the breadfruit. Using the retraced lines as a boundary, everything below it is erased as having been consumed" [11, p. 47]. The result is shown in Fig. 10.2 [13, p. 53]. In this case the retracing of some edges is part of the story. This clearly demonstrates the fact that backtracking is considered to be improper.

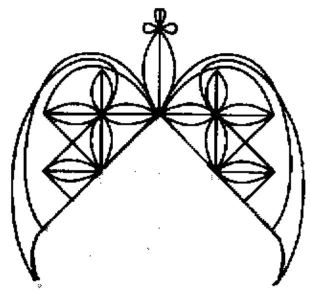

Fig. 10.2. Rat eats breadfruit, half remains

Our knowledge of the tracing of these figures is greatly indebted to the works of a young British ethnologist, Bernard Deacon, who spent two years in the Vanuatu islands in the early thirties, at the age of 22. He made decisive works concerning kinship, but he also recorded about one hundred sand drawings. What is important for our ethnomathematical studies is that Deacon not only reproduced the figures themselves in his field notes. He also had the intuition to record the exact tracing path of these figures. He did so by adding numbers to the edges of the figures, so that the sequence of numbered edges corresponds exactly to the tracing path. Unfortunately, Deacon died of blackwater fever as he was awaiting transportation home. His works are known thanks to the field notes that were found in his luggage. His annotated figures of sand drawings were published in 1934 [22].

The drawing of the tortoise is one of the most well-known and beautiful drawings of the Vanuatu tradition. If we analyse its tracing path as recorded by Deacon, we can find some regularities in it. The tracing is made of subgraphs with constraints similar to the whole graph itself. In addition to these intermediate stages, one finds connecting paths that combine more elementary shapes.

The tracing paths recorded by Deacon provide field materials of great interest for ethnomathematical investigations. In the next section, we shall study in detail a simple but quite ingenious geometric construction that can

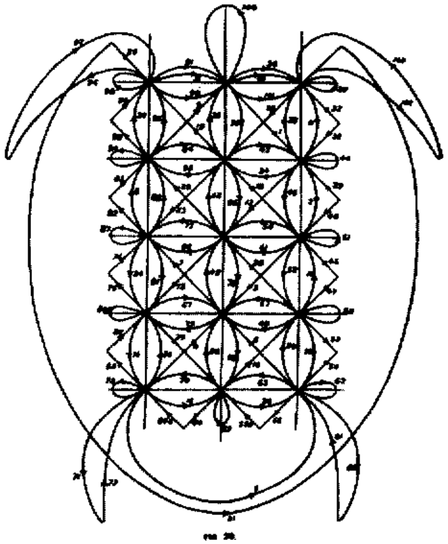

Fig. 10.3. The tortoise as recorded by Deacon

be deduced from these sand drawings. But this requires that we first travel to another part of the world, to Angola, where there is also a rich tradition of sand drawings.

10.1.2 The Logic of the Long Line

This tradition of sand drawings from the Tshokwe of Angola has also been studied from an ethnomathematical point of view by Paulus Gerdes, a mathematician from Mozambic, who wrote a book on the subject describing a lot of figures and studying their properties. The tracing of theses drawings obeys a rule similar to the one we have encountered in the Vanuatu tradition. Figures have to be drawn with a continuous path, the finger being kept in contact with the sand. In the Tshokwe tradition, this rule corresponds to what Gerdes calls the *monolinearity* property. It slightly differs from what mathematicians call a Eulerian path in that lines can cross one another, but are not allowed to touch each other without crossing [24, p. 20].

The drawing reproduced in Fig. 10.5 is similar to others one can find in Angola. Its shape looks like a lattice. The figure has 9 rows and 7 columns of points (with additional secondary columns and rows). Drawings similar to this one can be found in Angola, but they do not have the same number of rows and columns. The reason why is that when one tries to trace this specific figure with a continuous path satisfying the monolinearity property, the path joins its starting point before it can complete the figure.

What is interesting from an ethnomathematical point of view is that for particular lattices of points with numbers of rows and columns which do not

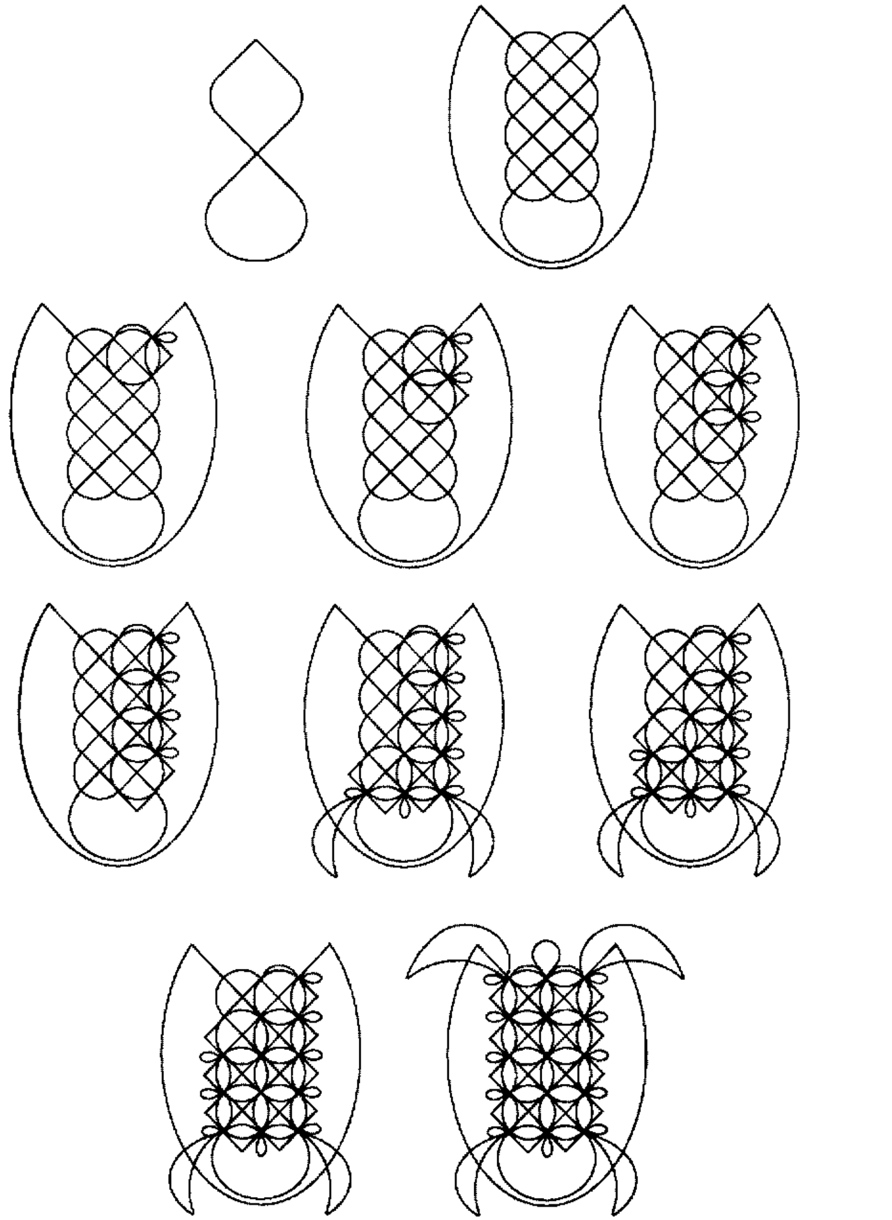

Fig. 10.4. The tortoise, intermediate stages of the tracing path

permit monolinear figures, the Tshokwe have discovered a geometric construction which transforms the figure into one which is monolinear although the numbers of rows and columns remain the same. This construction can be

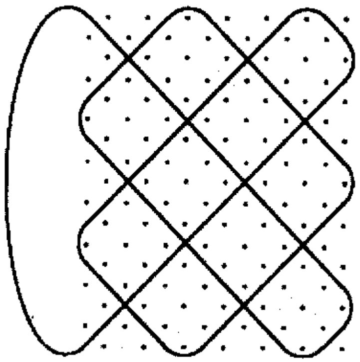

Fig. 10.5. A non-monolinear figure (lattice 9 × 7)

described as follows: a column is chosen, and each pair of crossing lines along this column is replaced by two quarter circles, as shown in Fig. 10.6.

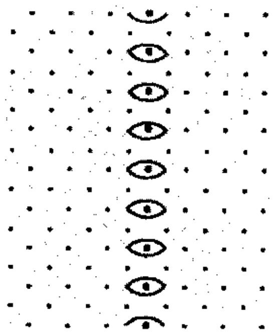

Fig. 10.6. A geometric algorithm

When one tries to draw the new figure, obtained by applying this transformation, the resulting path covers the entire figure. The figure produced by this algorithm is thus monolinear.

In Gerdes' book, this construction is presented as an "algorithm" [24, p. 205], that is a general method designed to transform lattice-like figures that are not monolinear into figures that are monolinear.

Going back to the Vanuatu, it is amazing that the geometric construction used by the artists from Angola seems to have been known by those from the Vanuatu. Among the figures recorded by Deacon, one can find the following one (Fig. 10.8), which is the same [22]. This one is entitled "Bird in the nest", as it represents a bird sitting on her eggs. As indicated by Deacon, the eggs correspond precisely to the ovals introduced by the algorithm. On the left

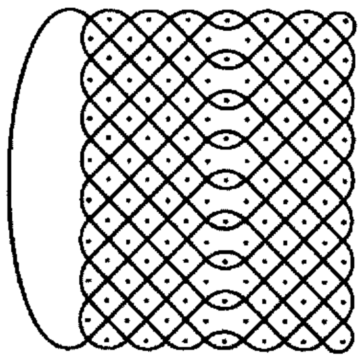

Fig. 10.7. The resulting monolinear figure

side is the (large) head of the bird. On the right side are the feathers of the tail, added as ornaments.

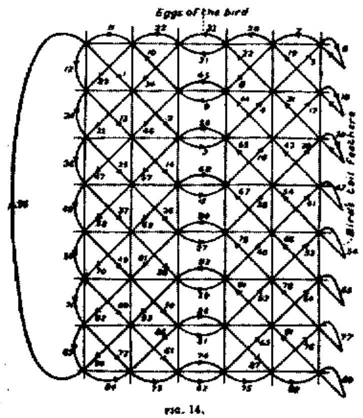

Fig. 10.8. Bird in the nest as recorded by Deacon (lattice 8 × 5)

As one can see in the examples of sand drawings discussed earlier, there are some clear indications establishing that the graph property of the figures is deliberately produced by the tracing path. It was explicitly mentioned in the figure entitled "Rat eats breadfruit, half remains" in Vanuatu, and this is also the case in Angola, as reported by Gerdes. Concerning the geometric construction that has been described in this section, one can verify that many drawings from Angola are based on the same principle, with different numbers of rows and columns, and that from a mathematical point of view, the construction has the property to transform non-monolinear figures into monolinear ones. Gerdes presents this construction as being a "very powerful algorithm" explicitly used by the native artists, but we do not know exactly what the artists have in mind when they draw these figures. To what extent are they conscious of the property of the construction? The level of cognition involved in this practice is not yet clear.

In the following two parts of my paper, we shall try to adapt the same approach to music. As we shall see, some amazing properties can be found in the musical examples which will be discussed, but the problem of identifying the correct cognitive level involved in the related mathematical constructions is much more difficult to resolve.

10.2 The Harp of the Former Nzakara Courts

10.2.1 The Art of Poet-Harpists

We now turn to Central Africa, where the Nzakara people live, in order to study some aspects of their harp music. Nzakara and Zande harps are well-known, because of the beautiful sculpted heads which adorn their necks. The Musée de la musique in Paris presented in 1999 an exhibition devoted to these instruments. While having fallen into neglect, these traditional instruments were still played upon request in 1993 by some old harp players, as one can hear by listening to the record entitled *Music from the former Bandia courts* published in the collection CNRS/Musée de l'homme founded by Gilbert Rouget [49]. The Nzakara-Zande territory is spread between Central African Republic, Congo (ex-Zaire) and Sudan. The Nzakara peoples are related to Zande, whose belief in witchcraft has triggered a strong philosophical discussion on "rationality", as we have recalled at the beginning of this paper.

The Nzakara harpists' repertoire is divided into categories. Each piece of poetry sung with the accompaniment of the harp relies on a short harp formula played repetitively as an *ostinato*. There are different types of formulas designated by terms such as *ngbakia, limanza, gitangi.* These terms also refer to traditional dances, the harp formulas being adapted from rhythms and musical elements borrowed from the dance repertoire played on the portable xylophone or the drum.

The formulas belonging to the categories *limanza* and *ngbakia* are adapted from the portable xylophone repertoire. In fact, one knows this repertoire of xylophones only through formulas that have been adapted to the harp, since the portable xylophone has now completely disappeared from the Nzakara region. We know the portable xylophone was an important instrument in the former Nzakara kingdom, as it was played by King Bangassou himself, but we have no recordings of its repertoire. The formula reproduced in Fig. 10.9 belongs to the category *limanza.* An example based on this formula can be heard on the CD *La parole du fleuve*, track 5 [49]. The piece is played by Maliba in 1969, and was recorded by Éric de Dampierre. We shall present some amazing properties of this apparently simple formula, studied in detail in [14,15].

The harp formula shown in Fig. 10.9 is interesting when one looks closely at its structure. The strings are plucked by pairs, one with another. The formula can thus be considered the superimposition of two melodic lines, one played with the left and one played with the right. Something remarkable appears when one compares the upper line with the lower line shifted six notes ahead: they are nearly identical. Strange enough, this is what is called a "canon" in Western classical music.

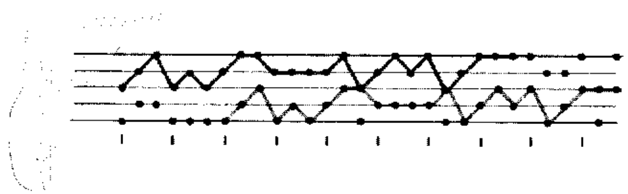

Fig. 10.9. A *limanza* canon: the two lines are nearly identical

Now let us go further into the analysis of this harp formula by describing an algorithm that can explain its construction. If one selects pairs of strings plucked simultaneously, taking the first one, then the sixth one, then the twelfth one and so on, up to five pairs denoted by numbers 0, 1, 2, 3, 4 (as shown in Fig. 10.10), the resulting sequence takes the form of a kind of ascending staircase.

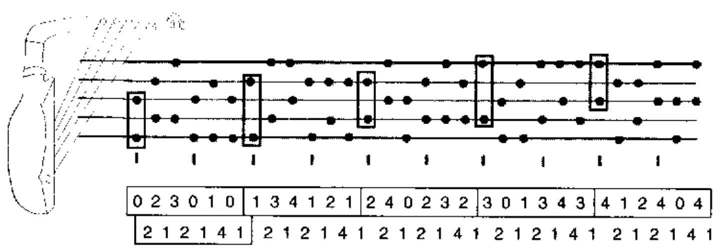

Fig. 10.10. Five pairs of strings plucked simultaneously

A property of the formula appears when one carries on this process by taking into account all the pairs occurring in the sequence. The formula being repeated as a loop, one can consider it an infinite periodic word over an alphabet, which is the finite cyclic group $\mathbf{Z}_5$. To such an infinite word, denoted by u, taking values into a finite cyclic group $\mathbf{Z}_p$, one can associate the *difference word* denoted by Du such that for every integer n, one has $Du(n) = u(n+1) - u(n)$. There are some interesting relations between the periodicity of the word u and the periodicity of the word Du, depending on the structure of the finite group $\mathbf{Z}_p$ [1]. In the case of Nzakara harp canons, the infinite words may be called *redundant*, since the period of Du is strictly less than the period of u. In fact, the word represented in Fig. 10.10 has a period of 30, whereas its associated difference word has a period of 6 (with values 2 1 2 1 4 1). This leads to an algorithm for generating harp canons. It suffices to consider a given word (for instance 0 2 3 0 1 0 in the formula represented in Fig. 10.10), and to translate it several times by adding the same value to its elements (for instance 1, which gives 1 3 4 1 2 1, then 2 4 0 2 3 2, and so on), until we reach the initial word again.

The interesting fact about this construction is that it is not limited to the harp formula shown in Fig. 10.10. It is a general construction that can be observed in different harp formulas of the Nzakara repertoire. Figure 10.11 shows another example, which belongs to the category called *ngbakia* (another category of music played formerly on the xylophone). This harp formula is built on the same principles. Here the initial sequence has only two pairs corresponding to 0 1, and the translation value is 3, which gives the resulting periodic word 0 1 3 4 1 2 4 0 2 3. We thus have more than one pattern illustrating the same construction, and the whole repertoire contains six such harp formulas. Hence, translating a given sequence by adding the same value to its elements appears to be a general algorithm applied in different contexts.

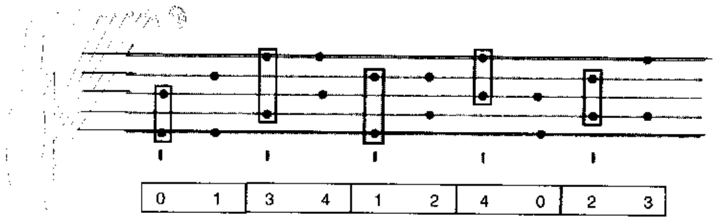

Fig. 10.11. A *ngbakia* harp formula

There is a surprising additional uniqueness property if one translates a word having only two pairs (as it is the case for the *ngbakia* formula shown in Fig. 10.11). Let us enumerate the different possible combinations, adding a few constraints to eliminate "degenerating" sequences. First, we demand that the resulting combination contain no repetition of a pair. Secondly, we demand that it not be factorised into two equal subsequences (these two conditions are motivated by the fact that harp formulas of the repertoire never fail to satisfy them). Having these conditions in mind, we enumerate all possible combinations obtained by translating different words containing two pairs. Since one can fix the initial pair to 0, there are only five possible values of the second pair, as shown in the table below.

The first combination is excluded, because it contains the repetition of a pair 0 0. The second is exactly the harp formula used by Nzakara harpists and shown in Fig. 10.11. The third is excluded, because it is a cyclic permutation of the previous one (as one can see by deleting its three initial elements 0 2 3). Since harp formulas are repeated endlessly, there is no way to

repetition	0 0 3 3 1 1 4 4 2 2
Nzakara solution	0 1 3 4 1 2 4 0 2 3
cyclic permutation of the solution	0 2 3 0 1 3 4 1 2 4
repetition	0 3 3 1 1 4 4 2 2 0
factorisation into two equal subsequences	0 4 3 2 1 0 4 3 2 1

distinguish two different cyclic permutations of the same formula by defining an initial point. Whereas there exist musical situations where the initial point of a sequence is a criteria for distinguishing different cyclic permutations (for instance, in ancient Greek theory of rhythm, "long-short" and "short-long" were considered distinct rhythms), no such reason exists in the repertoire of Nzakara harpists, so the second and third combinations are considered the same harp formula. The fourth combination is excluded, because it contains the repetition of a pair 3 3. The fifth combination is excluded because of its factorisation into two equal subsequences 0 4 3 2 1. The conclusion is then the uniqueness of the Nzakara solution.

10.2.2 The Plant-of-the-Twins

There is no way to connect the quite remarkable combinatorial properties of this harp formula to modes of thought of native people, since as we have already said, traditional harp music has fallen into neglect, and there are not enough Nzakara harpists still in activity to give us information about this question. All one can say is that according to the works of Éric de Dampierre, the musical structure of canon that has been discovered among those special harp formulas (in fact, there are six harp formulas which are canon in the same way as the previous example) could be related to some geometric considerations that Nzakara peoples make about a special plant used in the ritual for twins.

The plant called *bisibili* is represented in Fig. 10.12. As Éric de Dampierre has shown in his book *Penser au singulier*, this plant was of great importance in the old Nzakara tradition. It was the plant-of-the-twins, which means that when a pair of twins was born, a branch of it was planted in front of their house. It is interesting to point out that this plant was used in the ritual for twins for a purely "geometric" reason. What the Nzakara are interested in is the relative spatial positions of the two lines of leaves of the plant. These lines are not on the same plane. Furthermore, one of them is shifted so that the leaves are not attached to the stalk at the same points [19, p. 14].

There is a noticeable similarity between the shape of this plant and the musical structure of canon. The way one line of leaves is shifted along the stalk of the plant-of-the-twins could be related to the way the lower melodic line of harp formulas is shifted to produce a canon. Whereas we have no information that could establish this connection, we can assert that the ritual for twins reveals the existence of a clear interest for mathematical

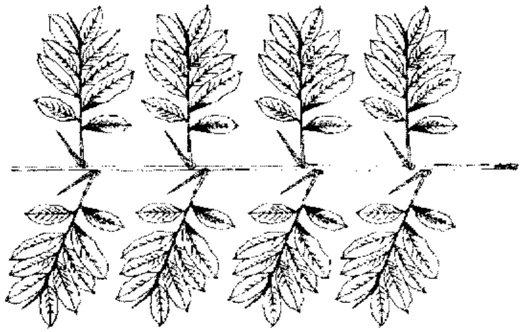

Fig. 10.12. The plant-of-the-twins (drawing by J.-M. Chavy)

ideas in the Nzakara tradition, involving geometric transformations such as glide-reflections. But the link between music and geometry has never been confirmed by native peoples. In a recent work on the same subject [12], Klaus-Peter Brenner has rejected an interpretation of the harp formulas as "canons". For various musico-cognitive considerations, he has developed a "permuting cell-sequencing structure", which is related to an algorithm based on the iterated translation of a given sequence.

10.3 African Asymmetric Rhythms (after the works of Simha Arom)

10.3.1 Asymmetric Rhythm of the Aka Pygmies

Our last section will be devoted to examples taken from the CD-ROM entitled *Pygmées Aka. Peuple et musique* realized by Simha Arom and his team [50]. These examples are four-part vocal polyphonies accompanied by a rhythmic combination the graphical representation of which is shown in Fig. 10.13. The corresponding piece of music, entitled *mbenzele*, can be heard in the section of the CD-ROM devoted to music analysis. We shall focus on a rhythmic pattern which is hidden in the rhythmic combination, but which can be played separately on the CD-ROM. This rhythmic pattern is marked with black squares in the graphical notation, Fig. 10.13. It is played with pairs of clashed metal blades called *diketo* [6, p. 438].

③ 2 2 2 2 ③ 2 2 2 2 2

Fig. 10.13. Aka Pygmies polyrhythm from the piece *mbenzele*

There is a kind of limp in this sequence, due to a shorter rhythmic duration inserted in the sequence of equal durations. This additional rhythmic value breaks the regularity of the sequence. Its duration is about half the duration of the one behind it. Bringing them together results in a duration equal to three units, marked with small rectangles in Fig. 10.13, whereas other durations are equal to two. Hence, the sequence as a whole can be viewed as groups of two-unit elements separated by a three-unit element.

This rhythmic pattern shares an interesting property of asymmetry. The three-unit elements of the sequence are not spaced in a strictly regular manner. The groups of two-unit elements, which separate them, contain four elements on one side, and five elements on the other side. Thus the pattern denoted as 3 2 2 2 2 3 2 2 2 2 2 contains two groups of unequal length. There exist other interesting asymmetric patterns of this type in traditional African music. They have been studied by Simha Arom who pointed out a specific property called the "rhythmic oddity" property ("imparité rythmique" in French) that will be presented in the next section. The following table shows all the patterns of this type that can be found in Arom's book [6], taken from various repertoires played in Central African Republic (Aka Pygmies p. 439 and 839, Zande repertoire of the *kponingbo* p. 470, Gbaya repertoire of the sanza, and Ngbaka repertoire of the harp p. 435 and 474).

3 3 2	Zande
3 2 3 2 2	Aka, Gbaya, Nzakara
3 2 2 3 2 2 2	Gbaya, Ngbaka
3 2 2 2 3 2 2 2 2	*not in use*
3 2 2 2 2 3 2 2 2 2 2	Aka

As one can see in the table above, one of the patterns is excluded (thus being written in italics in the table). It corresponds to value $k = 3$, where k and $k + 1$ denote the numbers of two-unit elements on both sides of the sequence, the only possible values for integer k being 0, 1, 2, and 4. The reason why $k = 3$ is not accepted is that patterns of the asymmetric type are always based on some regular pulse. One can note that this makes a great difference between them and other asymmetric rhythmic patterns called *aksak* used in Central Europe. African asymmetric rhythms are divided into two unequal parts, but are associated with a regular pulse, which can be grouped into two parts of equal length.

The numbers of beats in each pattern form a regular *geometric* progression 2, 4, 8, with no missing value. The corresponding numbers of units contained in each sequence depends on the division of the beat, which can be binary or ternary. The values for theses numbers are 8, 12, 16 and 24 (corresponding respectively to 2×4, 4×3, 4×4 and 8×3), and more generally, each number of units takes one of the possible two forms 2^a and $2^a \times 3$. The missing value

Fig. 10.14. Asymmetric African patterns and their corresponding pulse

in the preceding table is excluded, since it corresponds to a total number of units equal to 20.

10.3.2 The Rhythmic Oddity Property

In this last section, we shall be interested in some combinatorial properties of asymmetric patterns of the "rhythmic oddity" type. The rhythmic oddity property can be stated by placing the two- and three-unit elements of the sequence on a circle (thus expressing the fact that the pattern is played as a loop). For the Aka Pygmies sequence, denoted as 3 2 2 2 2 3 2 2 2 2 2, we get a three-unit element on top, a three-unit element on bottom, and groups of two-unit elements which separate them on both sides (four on the right side, and five on the left side). The property asserts that *if one attempts to break the circle into two parts, it is not possible to have two equal parts.* Whatever the chosen breaking point, there is always one unit lacking on one side.

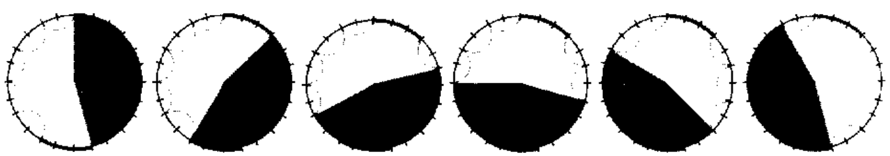

Fig. 10.15. Rhythmic oddity property: no breaking point giving two equal parts

This is what Simha Arom called the *rhythmic oddity* property. The asymmetry of the pattern is to some extent intrinsic, in the sense that there exists no breaking point giving two equal parts. Every division of the pattern gives two unequal parts, "half minus one" on the one side, and "half plus one" on the other side [7,8]. Note that the oddity property requires that the circle is divided into an even number of units, so that it is possible to find patterns of the "half minus one / half plus one" type. Denoting by n the total number

of units (n is even), a pattern can be viewed as an infinite n-periodic word on a two-letter alphabet, one letter corresponding to an attack and the other meaning "no attack". The rhythmic oddity property can then be stated as the fact that if there is an attack for integer i, then there must be no attack for integer $i + n/2$, a statement which means that no breaking point occurres in the sequence.

Now we turn to a combinatorial problem: is it possible to generate other patterns similar to the one in Fig. 10.15 by placing elements of three or two units on a circle, so that the resulting figure cannot be broken into two equal parts? Figure 10.16 shows the 24-unit circle, with two three-unit elements placed side by side. The resulting pattern does not satisfy the rhythmic oddity property, since it can be broken into two equal parts.

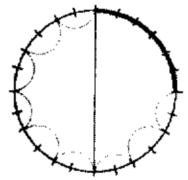

Fig. 10.16. Breakable pattern with three-unit elements placed side by side

It is easy to verify that there are only two possible ways to place the two three-unit elements on the circle, as shown in Fig. 10.17. But in fact, these two solutions are the same pattern, since they only differ by a cyclic permutation. Hence, there exists only one pattern with two elements of three units, which satisfies the oddity property, and this pattern is the one that was found in the Aka Pygmies repertoire.

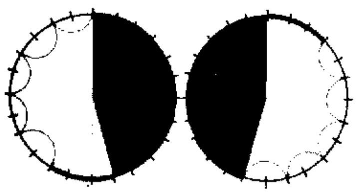

Fig. 10.17. Two ways of placing two elements of three units

If we look for patterns with more than two elements of three units, we must notice that the number of those elements must be even (if it is not, the total number of units cannot be even). Moreover, it can be proved that the number of two-unit elements must be odd. For instance, Fig. 10.18 shows a pattern where the numbers of elements of two and three units are equal to six and four, respectively. We have placed these elements on the circle in such a manner that the resulting pattern looks asymmetric. But although this pattern seems to satisfy the oddity property, there exists one point on the circle where it is possible to break it into two equal parts. This is a consequence of a simple argument related to the "pigeon hole principle" (the basic idea of Ramsey Theory asserting that n balls cannot be distributed into p boxes so that all boxes contain less than n/p balls) [25]. This argument implies that when the number of three-unit elements on one side of the circle is less than half the total number, then it must be greater than half this number on the other side. The initial breaking line, corresponding to $i = 0$, contains one element of three units on the right side, and three elements on the other side. When one rotates this line from point to point around the circle, the number of elements of three units contained in each side of the circle can be increased by one, decreased by one, or left unchanged. In the part of the circle which is initially on the right side, the number of elements of three units has grown from one to three as the breaking point is displaced halfway around the circle. Therefore, there exists an intermediate position where this number takes the value of two. This position is reached for $i = 4$, and at this point, we have two elements of three units on both sides of the circle. The number of remaining elements of length two being even, the circle can thus be broken into two parts of equal length. Finally, when the number of two- and three-unit elements are both even, the pattern cannot satisfy the oddity property. Since the number of three-unit elements must be even, as noticed above, it implies that the number of two-unit elements is odd.

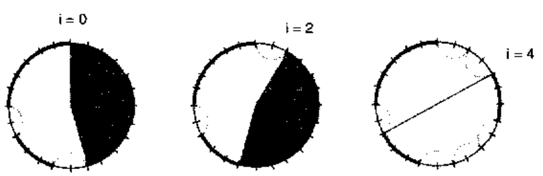

Fig. 10.18. Breakable pattern with four elements of three units

On the 24-unit circle, on can place only two, four or six elements of three units (recall that this number must be even). But the value four must be rejected, since in this case, the number of two-unit elements is also even. Indeed, denoting by n_2 and n_3 the numbers of two- and three-unit elements respectively, one has $2n_2 + 3n_3 = 24$, so that if $n_3 = 4$, then $n_2 = 6$ is even. The remaining values are then two and six. In the case of two elements of three units, we have found that there is only one solution, whatever being the total number of units on the circle, and the corresponding patterns are represented in Fig. 10.19.

In the case of six elements of three units (the total number of units on the circle being equal to 24), a computation has proved that there are only two patterns represented in Fig. 10.20.

One of them is split into two forms, a direct one and its mirror image:

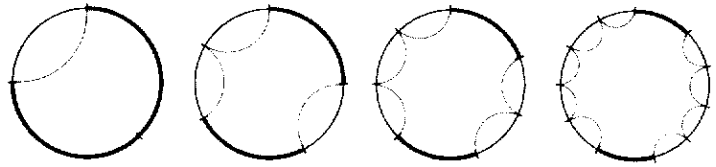

Fig. 10.19. All patterns with two elements of three units

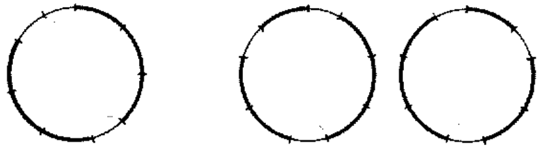

Fig. 10.20. All patterns with six elements of three units

pattern #1 = 3 3 3 2 3 3 3 2 2

pattern #2 = 3 3 3 2 3 3 2 3 2
mirror = 2 3 2 3 3 2 3 3 3

This abstract enumeration (done by computer) is obviously disconnected from the way native peoples think. But it is interesting to go back to their music with our generative result in mind to see what patterns are used in the repertoires of the Central African Republic. As we have already seen in the table of the previous section, all possible patterns involving two elements of length three are used. But what is much more surprising is that one of the two patterns involving six elements of length three (pattern #2) is also used. It contains groups of one, two and three elements of three units, separated by isolated elements of two units. This pattern is called *mokongo*, and it is used in the repertoire of the Aka Pygmies [6, p. 436].

The just explained *mokongo* is played in superimposition with the pattern we have already presented in the previous section (Fig. 10.13). The combination of the two is part of the piece *zoboko*, played during divination for the hunters (an example of which can be heard on the CD-ROM *Pygmées Aka*).

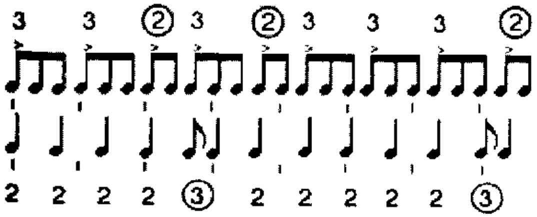

Fig. 10.21. Unbreakable pattern with six elements of three units

This example leads to a question related to perception. The rhythmic patterns shown in Fig. 10.21 are played in a fast tempo. The oddity property asserts that a pattern cannot be broken into two parts of equal length. But when one tries to break this pattern into two nearly equal parts, the length of those parts differ from half the total length of the pattern by just a single unit, which is a very short duration. It is impressive that such a short difference in duration is perceived so clearly that the oddity principle is universally observed in Pygmy music.

10.4 Conclusion

In both repertoires that we have studied, the formal structures are far too complex to be easily perceptible when one listens to the corresponding musical sequences. It is not realistic to assume that musical sequences of this form can be distinguished from others just by listening to them. Thus we have to admit that there might be a distance between musical forms produced by the human mind, and more elementary musical forms that human ear can identify. This statement is quite obvious in the case of Western classical music. It relies on the well-known fact that the score can be viewed from two different points of view, the composer's and the listener's (these points of view corresponding respectively to the "poïetique" and "esthesique" levels introduced in the French semiology of music, the score being associated to the "neutral" level) [35]. The composer's point of view must be distinguished from the listener's, because the richness of a musical masterpiece relies greatly on the fact that those points of view are not the same, which allows the listener to adopt many different points of view, each giving new qualities to the music. In the case of orally transmitted music traditions, this statement is less expected because no written score can be clearly identified with the neutral level, upon which the listener's multiple points of view are based. In some sense, Western classical music is closer to the Vanuatu sand drawing case. The figures traced in the sand could be to some extent associated with the neutral level. The fact that one of these figures can be traced using a continuous path is a property that is hardly recognizable when one looks at the figure. In this case, the formal property of the figure (the existence of a continuous path) is disconnected from qualities directly perceptible, and it remains a more speculative property.

As a last remark, I would like to go back to the cognitive issue addressed at the beginning of this paper. All the properties stated in this paper are related to visual or auditory forms produced by orally transmitted societies. None of them directly concerns the way these forms are produced, except to some extent the Eulerian property of sand drawings from the Vanuatu, which is mentioned explicitly in the mythology of the native peoples. From a cognitive point of view [3,4], our description focuses on the objects produced, and not on the cognitive processes producing them. When we describe what we call an "algorithm" producing an object a (this object being a lattice-like figure obtained by the geometric transformation we have studied in Angola, or a harp formula based on the translation of sequences in the Nzakara case), we construct a sequence of intermediate objects $a_0, a_1, \ldots$ with $a_n = a$. But this sequence of a_i can be viewed as an algorithm only if one can prove that the intermediate objects a_i have conscious existence in the native peoples' mind. Otherwise, we have only demonstrated a formal property of the given material.

In order to bring more cognitive insight to our investigations, we would need more information about the way people think about these objects and their related properties. Without such information, we only have a probabilistic understanding of the number of patterns satisfying some specific properties. Most of the properties we have studied were stated in combinatorial terms, by enumerating sets of combinations of elements, and by selecting only those satisfying specific properties. We have demonstrated that as soon as a formal property is discovered, the particular combinations satisfying this property were practiced quite generally. Among the countless number of possible combinations, those satisfying the given property that were used were more numerous than what would result from a random choice of these combinations (it was the case for the harp formulas built upon the translation of sequences, or the asymmetric rhythmic patterns satisfying the oddity property). There clearly exists a cognitive process underlying native people's choice, which favours this particular property, by selecting the corresponding patterns, whatever this selection process may be.

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References

1. M. Andreatta, D.T. Vuza: Tata Mountains Math. Publications **22** (to appear, 2001)
2. U. D'Ambrosio: ISGEm Newsletter, **4**, 1 (1988) pp. 5–8 (http://web.nmsu.edu/~pscott/isgem41.htm)
3. D. Andler (ed.): *Introduction aux sciences cognitives*, coll. Cerisy, Paris, Folio, 1992
4. D. Andler: 'Logique, raisonnement et psychologie', J. Dubucs, F. Lepage (eds.), *Méthodes logiques pour les sciences cognitives* (Hermès, Paris 1995) pp. 25–75

5. R. D'Andrade: *The Development of Cognitive Anthropology* (Cambridge University Press, 1995)
6. S. Arom: *Polyphonies et polyrythmies d'Afrique Centrale. Structure et méthodologie* (Selaf, Paris 1985) (English translation: *African polyphony and polyrhythm*, Cambridge, 1991)
7. S. Arom: 'Symétrie et ruptures de symétrie dans la musique de tradition orale: le cas de l'Afrique Centrale', *Quadrivium. Musiques et sciences* (Institut de pédagogie musicale et chorégraphique, Paris, 1992) pp. 209–215
8. S. Arom, J. Khalfa: Revue de musicologie, **84**, 1 (1998) pp. 5–17
9. M. & R. Ascher: 'Ethnomathematics', *History of Science*, xxiv (1986) pp. 125–144
10. M. Ascher: 'Graphs in cultures: a study in ethnomathematics', *Historia Mathematica*, xv (1988) pp. 201–227
11. M. Ascher: *Ethnomathematics. A multicultural view of mathematical ideas* (Chapman & Hall, New-York, 1991) (French translation by K. Chemla, S. Pahaut, *Mathématiques d'ailleurs*, Seuil, Paris, 1998)
12. K.P. Brenner: *Die kombinatorisch strukturierten Harfen- und Xylophonpattern der Nzakana (Zentralafrikanische Republik) als klingende Geometrie – eine Alternative zu Marc Chemilliers Kanonhypothese* (Holos-Verlag, Bonn, to appear)
13. J.-P. Cabane: *Ululan, les sables de la mémoire* (Grains de sable, Nouméa, 1997)
14. M. Chemillier: 'La musique de la harpe', E. de Dampierre (ed.), *Une esthétique perdue* (Presses de l'Ecole Normale Supérieure, Paris, 1995) pp. 99–208
15. M. Chemillier: 'Mathématiques et musiques de tradition orale', H. Genevois, Y. Orlarey (éds.), *Musique & Mathématiques* (Aléas-Grame, Lyon, 1997) pp. 133–143
16. M. Chemillier: *La logique de la longue ligne Vanuatu, conférence au Musée des Arts d'Afrique et d'Océanie*, 30 octobre 1997 (http://www.info.unicaen.fr/~marc/publi/vanuatu/ephemere.html)
17. M. Cole, J. Gay, J.A. Glick, D.W. Sharp: *Cultural Context of Learning and Thinking* (Basic Books, NY, 1971)
18. T. Crump: *Anthropology of numbers* (1990) (French translation by P. Lusson, *Anthropologie des nombres* (Seuil, Paris, 1995))
19. É. de Dampierre: *Penser au singulier* (Société d'ethnologie, Nanterre, 1984)
20. E. de Dampierre (ed.): *Une esthétique perdue* (Presses de l'Ecole Normale Supérieure, Paris, 1995)
21. P.J. Davis, R. Hersh: *The mathematical experience* (Birkhäuser, Boston, 1982) (French translation by L. Chambadal, *L'univers mathématique* (Gauthier-Villars, Paris, 1985))
22. B. Deacon: J. of the Royal Anthro. Institute, **64** (1934) pp. 129–175
23. E.E. Evans-Pritchard: Bull. of the Faculty of Arts, University of Egypt, Cairo, **ii**, 1 (1934) pp. 1–36
24. P. Gerdes: *Une tradition géométrique en Afrique. Les dessins sur le sable* (L'Harmattan, Paris, 1995)
25. R. Graham, B. Rotschild, J. Spencer: *Ramsey Theory* (Wiley, 1980)
26. J.L. Grootaers: *Witchcraft substance and "Zande logic"*, unpublished
27. J. Habermas: *Théorie de l'agir communicationnel*, tome 1: *Rationalité de l'agir et rationalisation de la société* (Fayard, Paris, 1987) (original edition 1981)
28. M. Hollis, S. Lukes (eds.): *Rationality and relativism* (Basil Blackwell, Oxford, 1982)

29. R. Horton (ed.): *La pensée métisse* (IUED, Genève, 1990)
30. G. Ifrah: *Histoire universelle des chiffres* (Seghers, Paris, 1981) (English translation *From One to Zero: A Universal History of Numbers* (Viking, New York, 1985)
31. P.N. Johnson-Laird, R.M.J. Byrne: *Deduction*, Essays in Cognitive Psychology (LEA, 1991)
32. I. Lakatos: *Proofs and Refutations* (Cambridge University Press, 1976) (French translation by N. Balacheff, J.-M. Laborde, *Preuves et réfutations* (Hermann, Paris, 1984))
33. L. Lévy-Brühl: Les *fonctions mentales dans les sociétés inférieures* (Paris, 1910) (English translation *How Natives Think* (George Allen and Unwin, London, 1926))
34. L. Lévy-Brühl: *La mentalité primitive* (Paris, 1922)
35. J.-J. Nattiez: *Fondements d'une sémiologie de la musique* (Paris, coll. 10/18, 1975) (English translation *Music and Discourse: Towards a Semiology of Music* (Princeton University Press, 1990))
36. J. Piaget, B. Inhelder: *La naissance de la pensée logique de l'enfance à l'adolescence* (1958) (English translation *The Growth of Logical Thinking from Childhood to Adolescence* (Routhledge and Kegan Paul, London, 1958))
37. G. Pòlya: Z. f. Kristal., lx (1924) pp. 278–282
38. J. Retschitzki: *Stratégies des joueurs d'awélé* (L'Harmattan, Paris, 1990)
39. A. Rosenfeld, R. Siromoney: Languages of design **1** (1993) pp. 229–245
40. A. Speiser: *Die Theorie der Gruppen von endlicher Ordnung*, 2e ed. (Springer, Berlin, 1927)
41. D. Sperber: *La contagion des idées* (Odile Jacob, 1996)
42. A. Szabo: *Les débuts des mathématiques grecques* (Vrin, Paris, 1977) (original edition Budapest, 1969)
43. E.B. Tylor: *Primitive Culture*, 2 vol. (London, 1871)
44. *Vanuatu. Océanie. Arts des îles de cendre et de corail*, catalogue de l'exposition, (Musée des Arts d'Afrique et d'Océanie, Paris, 1997)
45. H. Weyl: *Symétrie et mathématique moderne* (Flammarion, Paris, 1964)
46. B.R. Wilson (ed.): *Rationality* (Basil Blackwell, Oxford, 1970)
47. C. Zaslavsky: *Africa counts. Number and Pattern in African Culture* (Prindle, Weber & Schmidt, Boston, 1973) (French translation by V. Henderson (Ed. du Choix, Paris, 1995))

Audio CDs and CD-ROMs

48. *Central African Républic. Music of the former Bandia courts*, recordings, texts and photographs by M. Chemillier & E. de Dampierre, CNRS/Musée de l'Homme, Le Chant du Monde, CNR 2741009, Paris, 1996
49. *La parole du fleuve. Harpes d'Afrique Centrale*, Cité de la musique, CM001, Paris, 1999
50. *Pygmées Aka. Peuple et musique*, CD-ROM realized by S. Arom, S. Bahuchet, A. Epelboin, S. Fürniss, H. Guillaume, J. Thomas, Montparnasse Multimédia, Paris, 1998

10 Ethnomusicology, Ethnomathematics. The Logic Underlying Orally Transmitted Artistic Practices

Marc Chemillier

Ethnomathematics is a new domain that has arisen during the last two decades, at the crossroad between history of mathematics and mathematics education. This domain consists in the study of mathematical ideas shared by orally transmitted cultures. Such ideas are related to number, logic and spatial configurations [9,11]. My purpose is to show how ethnomusicology could turn musical materials in this direction. Music will be considered here as a mean of organizing time through patterns of sound events. Thus we shall focus on musical forms and structures, rather than on other aspects of music (such as social aspects for instance). We will ask whether particular forms of traditional music share specific properties, namely combinatorial properties, that could be of some interest from an ethnomathematical point of view.

The study of mathematical ideas of non-literate peoples goes against persisting notions in the mathematics literature, which are strongly influenced by the late nineteenth-century theory of classical evolution. According to this theory, cultures can be ordered on an intellectual scale from primitive peoples to Western culture [43]. These ideas have been quite influential in mathematics literature and continue to be cited [30]. Another idea developed by [33,34] introduced a distinction between the Western mode of thought, and a "prelogical" mode of thought (or "unscientific" as Evans-Pritchard said) characterizing traditional peoples. A rich debate has arisen from this controversial theory, involving anthropologists, cognitive psychologists, and philosophers, on the nature of "rationality", with special attention paid to the witchcraft problem of the Zande peoples from Sudan [23,46,28,29,26], with echoes in [27,31] and others. Ethnomathematics has grown in the wake of this epistemological debate, gathering mathematicians from different parts of the world including the southern hemisphere. A research program has been sketched, and an International Study Group was founded [2].

The efforts made by ethnomathematicians in order to correct erroneous theories on the ability of human thought to think abstractly or logically rely greatly on the works of former ethnologists who have recorded information involving mathematical ideas while doing field work at the end of the nineteenth or during the twentieth century. Not being especially engaged with mathematics in their own culture, these ethnologists did not extract the whole mathematical content of their recorded material. Thus a great amount of work remains in the study of this field material from a mathematical point of view. In the case of music, recorded material will consist in various forms of