

General Mathematics and Computational Science II

Exercise 14

April 12, 2007

1. Find all solutions for the underdetermined linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 3 & 3 & 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ -2 \\ 0 \end{pmatrix}.$$

2. Write the following linear programming problem in its standard form.

Maximize

$$z = 2x_1 - x_2 + x_3$$

subject to

$$x_1 - x_2 \leq 1,$$

$$x_2 - x_3 \geq 1,$$

$$x_3 - x_1 \leq 3,$$

$$x_1 \geq -2,$$

$$x_2 \leq 1,$$

$$x_3 \leq 0.$$

3. Assume that $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ are two solutions to the linear programming problem in standard form,

$$\text{minimize } z = \mathbf{c}^T \mathbf{x}$$

$$\text{subject to } A\mathbf{x} = \mathbf{b}$$

$$\text{and } \mathbf{x} \geq \mathbf{0}.$$

Show that any convex combination $\mathbf{z} = t\mathbf{x} + (1-t)\mathbf{y}$ for $t \in [0, 1]$ is also a solution.