

# General Mathematics and Computational Science II

## Exercise 16

April 19, 2007

1. Show that the leaving variable in one iteration of the simplex method can never be the entering variable in the next iteration.
2. The *primal* form of a linear programming problem is

$$\begin{aligned} & \text{minimize } \mathbf{c}^T \mathbf{x} \\ & \text{subject to } A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0. \end{aligned} \tag{P}$$

The corresponding dual problem reads

$$\begin{aligned} & \text{maximize } \mathbf{y}^T \mathbf{b} \\ & \text{subject to } \mathbf{y}^T A \leq \mathbf{c}^T. \end{aligned} \tag{D}$$

There  $A$  is an  $m \times n$  matrix,  $\mathbf{x}, \mathbf{c} \in \mathbb{R}^n$ , and  $\mathbf{y}, \mathbf{b} \in \mathbb{R}^m$ .

Show that if  $\mathbf{x}$  solves (P) and  $\mathbf{y}$  solves (D), then

$$\mathbf{y}^T \mathbf{b} \leq \mathbf{c}^T \mathbf{x}.$$

3. In the notation of the previous question, show that if  $\mathbf{x}$  is feasible for problem (P) and  $\mathbf{y}$  is feasible for problem (D), and if furthermore

$$\mathbf{y}^T \mathbf{b} = \mathbf{c}^T \mathbf{x},$$

then  $\mathbf{x}$  solves (P) and  $\mathbf{y}$  solves (D).