General Mathematics and Computational Science II

Exercise 21

May 10, 2007

1. Let G be a finite abelian group, written additively, and let $\hat{G}$ denote its dual group. Show that for every nonzero $a \in G$,

$$\sum_{\chi \in \hat{G}} \chi(a) = 0\,.$$

   Hint: Mimic the proof of the corresponding statement on G as given in class.

2. Prove the Plancharel identity

$$\langle \hat{f}, \hat{g} \rangle = n \, \langle f, g \rangle \,,$$

   with the inner products

$$\langle f, g \rangle = \frac{1}{n} \sum_{a \in G} \overline{f(a)} \, g(a) \,,$$
$$\langle \hat{f}, \hat{g} \rangle = \frac{1}{n} \sum_{\chi \in \hat{G}} \overline{\hat{f}(\chi)} \, \hat{g}(\chi) \,,$$

   and Fourier transform

$$\hat{f}(\chi) = \sum_{a \in G} \chi(a) \, f(a)\,.$$

3. Let $f_A \in \mathbb{C}^G$ denote the characteristic function of a set $A \subset G$, and $\hat{f}_A \in \mathbb{C}^{\hat{G}}$ its Fourier transform. Set

$$\Phi(A) = \max\{|\hat{f}_A(\chi)| \colon \chi \in \hat{G}, \chi \neq \chi_0\}\,.$$

   Show that $\Phi(A) = \Phi(G \setminus A)$.