General Mathematics and Computational Science I

Exercise 2

September 6, 2007

1. Use induction to show that for $n \in \mathbb{N} \cup \{0\}$ and any (real) number $q \neq 1$,

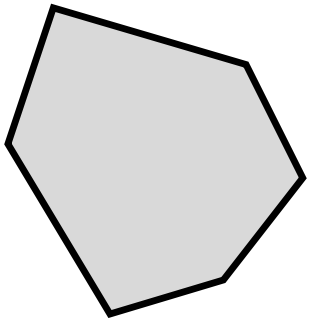
$$\sum_{k=0}^{n} q^k = \frac{1 - q^{n+1}}{1 - q}\,.$$

   Remark: This sum is called a *geometric progression*.

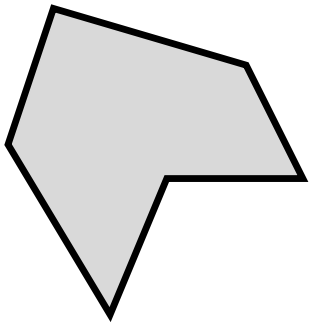
2. A polygon is called *convex* if any straight line connecting two points of the polygon lies entirely within; see figure below.

   Use induction to show that, for a convex n-gon (a polygon with n vertices), the sum of the interior angles S_n is

$$S_n = 180 \cdot (n - 2)$$



Convex polygon



Non−convex polygon

3. (From Daepp/Gorkin, p. 114) Let X be a nonempty set with an equivalence relation $\sim$ on it. Prove that for all $x, y \in X$,

$$[x] = [y] \qquad \text{if and only if} \qquad x \sim y\,.$$