General Mathematics and Computational Science I

Exercise 6

September 20, 2007

1. (Bernoulli inequality.) Use induction to prove that for every $x \in \mathbb{Q}$ with $x \geq -1$ and every $n \in \mathbb{N}$,
$$(1+x)^n \geq 1 + n\,x\,.$$

2. Recall from class that two sets A and B are said to be *isomorphic*, written $A \cong B$, if there exists a bijective map $\phi\colon A \to B$.

   Show that the relation "$\cong$" is in fact an equivalence relation.

3. We say a field $\mathbb{F}$ is *ordered* if there exists a relation $>$ which respects addition and multiplication in the following way. For any $x, y, z \in \mathbb{F}$,

   (O1) if $x > y$, then $x + z > y + z$;

   (O2) if $x > y$ and $z > 0$, then $xz > yz$.

   Recall from class that the set of rational numbers $\mathbb{Q}$ are equivalence classes $[a/b]$ with respect to the equivalence relation $(a,b) \sim (a',b')$ if $a \cdot b' = a' \cdot b$. Recall that addition, multiplication, and ordering of two rationals $x = [a/b] \in \mathbb{Q}$ and $y = [c/d] \in \mathbb{Q}$ are defined as follows.

   (a) $x + y = [(ad + bc)/bd]$

   (b) $x \cdot y = [ac/bd]$

   (c) x is *positive* if $ab > 0$ and *negative* if $ab < 0$; $x > y$ if $x - y$ is positive.

   Show that this order relation satisfies (O1) and (O2).