

General Mathematics and Computational Science I

Exercise 8

September 27, 2007

1. A company has $2n$ employees.
 - (a) In how many ways can they be divided into n groups of two members each?
 - (b) In how many ways can they be divided into two departments of n members each, if each department must choose a president and a vice-president?
2. (From Ivanov, p. 17.)
 - (a) Verify, by explicit computation, that the binomial coefficients

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

satisfy the recursion relation

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}. \quad (*)$$

- (b) Derive this recursion relation from the interpretation of the binomial coefficients as the “number of k -element subsets of an n -element set”.
3. (From Ivanov, p. 19.) Prove that

$$\binom{2n}{k} = \sum_{l=0}^k \binom{n}{l} \binom{n}{k-l}.$$

Hint: Use the function $P_n(x)$ from class, and the fact that

$$((1+x)^n)^2 = (1+x)^{2n}.$$