General Mathematics and Computational Science I

Exercise 11

October 18, 2007

1. Show that
$$\sum_{k=0}^{n} \binom{n+k}{k} \frac{1}{2^k} = 2^n$$
Hint: Denote the left hand side by $f(n)$ and prove that $f(n+1) = 2\,f(n)$.

2. Generalized binomial coefficients can be defined as the coefficients in the formal power series
$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k \,.$$
Show that
$$\binom{\alpha}{k+1} = \frac{\alpha-k}{k+1} \binom{\alpha}{k}$$
for every real number α, and $0 \leq k$.

3. Use the method of generating functions to find a closed form expression for the members of the sequence
$$c_0 = 1 \,,$$
$$c_{n+1} = \sum_{k=0}^{n} c_k \, c_{n-k} \,.$$
Hint: Your answer will involve generalized binomial coefficients with $\alpha = \frac{1}{2}$, see Question 2 above. You may leave your answer in terms of these coefficients; there is no need to further expand the expressions although you may want to check the first couple of terms to see whether your answer is correct.

4. (*From previous homework.*) Use the method of generating functions to find a closed form expression for the members of the generalized Fibonacci sequence
$$a_0 = A \,,$$
$$a_1 = B \,,$$
$$a_n = a_{n-1} + a_{n-2} \,.$$