General Mathematics and Computational Science I

Exercise 14

October 30, 2007

1. Solve Problem 14 from Ivanov, p. 50.

2. For positive real numbers $a_1, \ldots, a_n$, let

$$G(a_1, \ldots, a_n) = \left(\prod_{i=1}^{n} a_i \right)^{\frac{1}{n}}$$

denote their geometric mean, and

$$A(a_1, \ldots, a_n) = \frac{1}{n} \sum_{i=1}^{n} a_i$$

their algebraic mean. Show that, if $a_1 \neq a_2$, then

$$G(a_1, \ldots, a_n) < G(a^*, a^*, a_3, \ldots, a_n)$$

and

$$A(a_1, \ldots, a_n) = A(a^*, a^*, a_3, \ldots, a_n) \,,$$

where

$$a^* = \frac{a_1 + a_2}{2} \,.$$