

# General Mathematics and Computational Science I

## Exercise 15

November 1, 2007

1. Solve Problem 17 from Ivanov, p. 53.
2. (From Ivanov, p. 53) Show that

$$\sqrt[n]{n} > \sqrt[n+1]{n+1}$$

for  $n \geq 3$ .

*Hints:*

- (a) Consider the expression  $(1+n)^n n^{\frac{n+1}{n}}$ , use the AM-GM inequality;
- (b) Notice that

$$3^{-\frac{1}{3}} + \frac{1}{4} \approx 0.943361 < 1;$$

- (c) part of the argument is inductive.

(Extra points if you find a proof which is more direct than the one suggested by the hints above.)

3. Prove *Jensen's inequality*

$$\left( \sum_{i=1}^n a_i^p \right)^{\frac{1}{p}} \geq \left( \sum_{i=1}^n a_i^q \right)^{\frac{1}{q}}$$

whenever  $0 < p \leq q$  and  $a_1, \dots, a_n \geq 0$ . If  $p < q$ , then equality occurs if and only if at most one of the  $a_i$  is nonzero.

*Hint:* A proof is sketched on p. 57 in Ivanov. You should fill in the gaps and write out a complete, self-contained proof.