General Mathematics and Computational Science I

Exercise 16

November 6, 2007

1. Check the conditions for Laplace's method for the function

$$f(x) = x - \ln x - 1$$

which appears in the proof of Stirling's formula. I.e., check the following.

   (a) f is strictly decreasing for $0 < x < 1$, strictly increasing for $x > 1$, and $f(1) = 0$.

   (b) There are positive constants b and c such that $f(x) \geq b\,x$ for $x \geq c$.

   (c) $f(x) = a\,(x-1)^2 + \psi(x)\,(x-1)^3$ where ψ is a bounded function for $x \in [1-\delta, 1+\delta]$ for some $\delta > 0$. How must a be chosen?

   Hint: For (c), use either Taylor's formula with remainder (easy), or l'Hôpital's rule for the function

$$g(x) = \frac{f(x)}{(x-1)^2}\,.$$

   (more elementary, but longer).

2. Use Stirling's formula to prove that

$$\frac{n!}{(n/2)!^2} \sim 2^n \sqrt{\frac{2}{\pi n}}\,.$$