

General Mathematics and Computational Science I

Exercise 19

November 22, 2007

1. The population of some species is modeled by

$$x_{n+1} = \mu x_n e^{-x_n}, \quad (1)$$

with $\mu > 0$.

- (a) Prove that $x^* = \ln \mu$ is the only equilibrium point.
 - (b) For which values of μ is the equilibrium point asymptotically stable, for which values is it unstable?
2. For a given continuously differentiable function g , consider the difference equation

$$x_{n+1} = x_n + h g(x_n).$$

Determine conditions on h for which an equilibrium point is asymptotically stable, respectively unstable.

Remark: This difference equation is in fact the so-called Euler method for solving the differential equation $\dot{y} = g(y)$. However, this is not relevant for answering this question.

3. Show that the logistic map

$$x_{n+1} = \mu x_n (1 - x_n)$$

has a 2-cycle whenever $\mu > 3$ with

$$a = \frac{1 + \mu - \sqrt{(\mu - 3)(\mu + 1)}}{2\mu},$$
$$b = \frac{1 + \mu + \sqrt{(\mu - 3)(\mu + 1)}}{2\mu}.$$