

# General Mathematics and Computational Science I

## Exercise 20

November 27, 2007

1. On the previous homework, you have shown that the logistic map

$$x_{n+1} = \mu x_n (1 - x_n)$$

has a 2-cycle whenever  $\mu > 3$  with

$$a = \frac{1 + \mu - \sqrt{(\mu - 3)(\mu + 1)}}{2\mu},$$
$$b = \frac{1 + \mu + \sqrt{(\mu - 3)(\mu + 1)}}{2\mu}.$$

Show that this 2-cycle is asymptotically stable for  $3 < \mu < 1 + \sqrt{6}$  and unstable for  $\mu > 1 + \sqrt{6}$ .

2. Solve the so-called *Pielou logistic equation*

$$x_{n+1} = \frac{\alpha x_n}{1 + \beta x_n},$$

find its equilibrium points and determine their stability.

*Hint:* To solve the equation, substitute  $x_n = 1/y_n$ .