

# Applied Analysis

## Homework 4

due October 19, 2007

1. Use the method of steepest descent to determine the first two terms in an asymptotic expansion of

$$I(\lambda) = \int_0^{\pi/4} \cos(\lambda z^2) \tan^2 z \, dz$$

as  $\lambda \rightarrow \infty$  from  $\mathbb{R}_+$ .

2. Consider the integral

$$I(\lambda) = \int_0^1 e^{-4\lambda z^2} \cos(5\lambda z - \lambda z^3) \, dz$$

as  $\lambda \rightarrow \infty$  from  $\mathbb{R}_+$ .

- (a) Explain why not even the leading order of its asymptotic expansion can be found by Laplace's method with  $\Phi(z) = -4z^2$  and  $g(z) = \cos(5\lambda z - \lambda z^3)$ .
- (b) Show that

$$I(\lambda) = \frac{1}{2} e^{-2\lambda} \left( \sqrt{\frac{\pi}{\lambda}} + O(\lambda^{-3/2}) \right)$$

by using a steepest-descent path of integration in the complex plane.

3. Miller, p. 110, Exercise 4.3.
4. Miller, p. 110, Exercise 4.4.