

# Applied Analysis

## Homework 6

due November 9, 2007

1. Miller, p. 181, Exercise 5.18.
2. Consider the equation

$$\begin{aligned}y''(x) + (k^2 + \varepsilon \mu(x)) y(x) &= 0, \\y(0) &= 1, \\y'(0) &= 0.\end{aligned}$$

The first order regular perturbation approximation was computed in class (or in Miller, pp. 258–260).

- (a) What is the interval of validity of the approximation which is guaranteed by the abstract justification argument (as done in class or in Miller, Section 7.1.3)?
- (b) Is this interval long or short as compared to the growth rate of the secular term in the case of parametric resonance?
- (c) Investigate the full equation and its first order regular perturbation approximation numerically (e.g. using *Mathematica*). How should you quantify the accuracy of the approximation? What can you say about the interval of validity of the asymptotic approximation?
- (d) Does the true solution exhibit secular growth beyond the interval of validity of the first order approximation? Can you provide a rigorous analysis?