Partial Differential Equations

Homework 3

due September 26, 2008

1. Let
$$K(x,y) = \frac{2}{n\alpha(n)} \frac{x_n}{|x-y|^n}$$
be the Poisson kernel for the half-plane. Show that
$$\int_{\partial \mathbb{R}^n_+} K(x,y)\,dy = 1$$
for any $x \in \mathbb{R}^n_+$.

2. In class we have used Liouville's theorem to show that any bounded solution of the Poisson equation
$$-\Delta u = f$$
for $f \in C^2_{\mathrm{c}}(\mathbb{R}^n)$, $n \geq 3$, is given by the solution formula
$$u(x) = \int_{\mathbb{R}^n} \Phi(x-y)\,f(y)\,dy$$
up to an arbitrary additive constant. (Evans, p. 30, Theorem 8.)

   This statement does not hold in dimension $n = 2$ since solutions are generically unbounded. Use Liouville's theorem to conjecture and prove the corresponding theorem for $n = 2$.

3. Evans, p. 86 problem 5

4. Evans, p. 86 problem 6