Partial Differential Equations

Homework 4

due November 12, 2009

1. We have shown in class that if $u \in L^2((0,T);H_0^1(U))$ and $u' \in L^2((0,T);H^{-1}(U))$, then
$$u \in C([0,T];L^2(U))$$
and the map $t \mapsto \|u(t)\|^2_{L^2(U)}$ is absolutely continuous with
$$\frac{d}{dt}\|u(t)\|^2_{L^2(U)} = 2\,\langle u'(t), u(t)\rangle_{H^{-1},H_0^1}$$
for a.e. $t \in [0,T]$.
Show that, moreover, there exists a constant $c = c(T)$ such that
$$\|u\|_{C([0,T];L^2(U))} \leq c\left(\|u\|_{L^2((0,T);H_0^1(U))} + \|u'\|_{L^2((0,T);H^{-1}(U))}\right).$$

2. Let $\{w_k\} \subset H_0^1(U)$ be an orthonormal basis of $L^2(U)$. Show that the set of functions of the form
$$v = \sum_{k=1}^{N} d_k(t)\,w_k\,,$$
where the d_k are smooth functions of t, is dense in $L^2((0,T);H_0^1(U))$.

3. Evans, p. 425, Problem 1

4. Evans, p. 425, Problem 2

5. Evans, p. 425, Problem 3

6. Evans, p. 425, Problem 4