25. $\lim_{x\to 2} \dfrac{x+2}{(x-2)^2}$

26. $\lim_{x\to 1^-} \dfrac{x}{x-1}$

27. $\lim_{x\to 3^+} \dfrac{x}{x-3}$

28. $\lim_{x\to 1^-} \dfrac{x-2}{x^2-3x+2}$

29. $\lim_{x\to 1^-} \dfrac{x+1}{(x-1)^3}$

30. $\lim_{x\to 5^+} \dfrac{25-x^2}{x^2-10x+25}$

31. $\lim_{x\to 0} \dfrac{\sin 3x}{x}$

32. $\lim_{x\to 0} \dfrac{\tan 5x}{x}$

33. $\lim_{x\to 0} \dfrac{\sin 3x}{\sin 2x}$

34. $\lim_{x\to 0} \dfrac{\tan 2x}{\tan 3x}$

35. $\lim_{x\to 0^+} \dfrac{x}{\sin \sqrt{x}}$

36. $\lim_{x\to 0} \dfrac{1-\cos 3x}{2x}$

37. $\lim_{x\to 0} \dfrac{1-\cos 3x}{2x^2}$

38. $\lim_{x\to 0} x^3 \cot x \csc x$

39. $\lim_{x\to 0} \dfrac{\sec 2x \tan 2x}{x}$

40. $\lim_{x\to 0} x^2 \cot^2 3x$

In Problems 41 through 46, apply your knowledge of lines tangent to parabolas (Section 2.1) to write a slope-predictor formula for the given curve $y = f(x)$. Then write an equation for the line tangent to $y = f(x)$ at the point $(1, f(1))$.

41. $f(x) = 3 + 2x^2$

42. $f(x) = x - 5x^2$

43. $f(x) = 3x^2 + 4x - 5$

44. $f(x) = 1 - 2x - 3x^2$

45. $f(x) = (x-1)(2x-1)$

46. $f(x) = \dfrac{x}{3} - \left(\dfrac{x}{4}\right)^2$

In Problems 47 through 53, use the "four-step process" of Section 2.2 to find a slope-predictor formula for the graph $y = f(x)$.

47. $f(x) = 2x^2 + 3x$

48. $f(x) = x - x^3$

49. $f(x) = \dfrac{1}{3-x}$

50. $f(x) = \dfrac{1}{2x+1}$

51. $f(x) = x - \dfrac{1}{x}$

52. $f(x) = \dfrac{x}{x+1}$

53. $f(x) = \dfrac{x+1}{x-1}$

54. Find a slope-predictor formula for the graph

$$f(x) = 3x - x^2 + |2x + 3|$$

at the points where a tangent line exists. Find the point (or points) where no tangent line exists. Sketch the graph of f.

55. Write equations of the two lines through $(3, 4)$ that are tangent to the parabola $y = x^2$. (*Suggestion:* Let (a, a^2) denote either point of tangency; first solve for a.)

56. Write an equation for the circle with center $(2, 3)$ that is tangent to the line with equation $x + y + 3 = 0$.

In Problems 57 through 60, explain why each function is continuous wherever it is defined by the given formula. For each point a where f is not defined by the formula, tell whether a value can be assigned to $f(a)$ in such a way to make f continuous at a.

57. $f(x) = \dfrac{1-x}{1-x^2}$

58. $f(x) = \dfrac{1-x}{(2-x)^2}$

59. $f(x) = \dfrac{x^2+x-2}{x^2+2x-3}$

60. $f(x) = \dfrac{|x^2-1|}{x^2-1}$

61. Apply the intermediate value property of continuous functions to prove that the equation $x^5 + x = 1$ has a solution.

62. Apply the intermediate value property of continuous functions to prove that the equation $x^3 - 4x^2 + 1 = 0$ has three different solutions.

63. Show that there is a number x between 0 and $\pi/2$ such that $x = \cos x$.

64. Show that there is a number x between $\pi/2$ and π such that $\tan x = -x$. (*Suggestion:* First sketch the graphs of $y = \tan x$ and $y = -x$.)

65. Find how many straight lines through the point $(12, \frac{15}{2})$ are normal to the graph of $y = x^2$ and find the slope of each. (*Suggestion:* The cubic equation you should obtain has one root evident by inspection.)

66. A circle of radius r is dropped into the parabola $y = x^2$. If r is too large, the circle will not fall all the way to the bottom; if r is sufficiently small, the circle will touch the parabola at its vertex $(0, 0)$ (see Fig. 2.4.22). Find the largest value of r so that the circle will touch the vertex of the parabola.

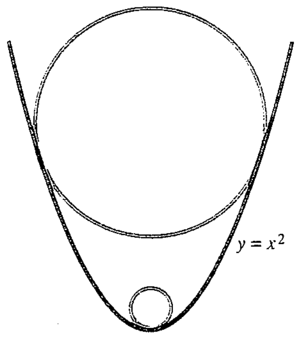

Fig. 2.4.22 If the circle is too large, it cannot touch the bottom of the parabola (Problem 66).

Apply the limit laws to evaluate the limits in Problems 1 through 40 or to show that the indicated limit does not exist, as appropriate.

1. $\lim_{x\to 0} (x^2 - 3x + 4)$

2. $\lim_{x\to -1} (3 - x + x^3)$

3. $\lim_{x\to 2} (4 - x^2)^{10}$

4. $\lim_{x\to 1} (x^2 + x - 1)^{17}$

5. $\lim_{x\to 2} \dfrac{1+x^2}{1-x^2}$

6. $\lim_{x\to 3} \dfrac{2x}{x^2-x-3}$

7. $\lim_{x\to 1} \dfrac{x^2-1}{1-x}$

8. $\lim_{x\to -2} \dfrac{x+2}{x^2+x-2}$

9. $\lim_{t\to -3} \dfrac{t^2+6t+9}{9-t^2}$

10. $\lim_{x\to 0} \dfrac{4x-x^3}{3x+x^2}$

11. $\lim_{x\to 3} (x^2-1)^{2/3}$

12. $\lim_{x\to 2} \sqrt{\dfrac{2x^2+1}{2x}}$

13. $\lim_{x\to 3} \left(\dfrac{5x+1}{x^2-8}\right)^{3/4}$

14. $\lim_{x\to 1} \dfrac{x^4-1}{x^2+2x-3}$

15. $\lim_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7}$

16. $\lim_{x\to 1^+} (x - \sqrt{x^2-1})$

17. $\lim_{x\to -4} \dfrac{\dfrac{1}{\sqrt{13+x}} - \dfrac{1}{3}}{x+4}$

18. $\lim_{x\to 1^+} \dfrac{1-x}{|1-x|}$

19. $\lim_{x\to 2^+} \dfrac{2-x}{\sqrt{4-4x+x^2}}$

20. $\lim_{x\to -2^-} \dfrac{x+2}{|x+2|}$

21. $\lim_{x\to 4^+} \dfrac{x-4}{|x-4|}$

22. $\lim_{x\to 3^-} \sqrt{x^2-9}$

23. $\lim_{x\to 2^+} \sqrt{4-x^2}$

24. $\lim_{x\to -3} \dfrac{x}{(x+3)^2}$