By using a graphics calculator or computer with graphics capabilities, you can verify that the graph of the function f of Example 6 is that shown in Fig. 3.5.11. But in the usual case of a continuous function that has only finitely many critical points in a given closed interval, the closed-interval maximum-minimum method suffices to determine its maximum and minimum values without requiring any detailed knowledge of the graph of the function.

3.5 PROBLEMS

In Problems 1 through 10, state whether the given function attains a maximum value or a minimum value (or both) on the given interval. (*Suggestion:* Begin by sketching a graph of the function.)

1. $f(x) = 1 - x;\ [-1, 1)$ **2.** $f(x) = 2x + 1;\ [-1, 1)$

3. $f(x) = |x|;\ (-1, 1)$ **4.** $f(x) = \dfrac{1}{\sqrt{x}};\ (0, 1]$

5. $f(x) = |x - 2|;\ (1, 4]$

6. $f(x) = 5 - x^2;\ [-1, 2)$

7. $f(x) = x^3 + 1;\ [-1, 1]$

8. $f(x) = \dfrac{1}{x^2 + 1};\ (-\infty, \infty)$

9. $f(x) = \dfrac{1}{x(1 - x)};\ [2, 3]$

10. $f(x) = \dfrac{1}{x(1 - x)};\ (0, 1)$

In Problems 11 through 40, find the maximum and minimum values attained by the given function on the indicated closed interval.

11. $f(x) = 3x - 2;\ [-2, 3]$

12. $f(x) = 4 - 3x;\ [-1, 5]$

13. $h(x) = 4 - x^2;\ [1, 3]$

14. $f(x) = x^2 + 3;\ [0, 5]$

15. $g(x) = (x - 1)^2;\ [-1, 4]$

16. $h(x) = x^2 + 4x + 7;\ [-3, 0]$

17. $f(x) = x^2 - 3x;\ [-2, 4]$

18. $g(x) = 2x^3 - 9x^2 + 12x;\ [0, 4]$

19. $h(x) = x + \dfrac{4}{x};\ [1, 4]$

20. $f(x) = x^2 + \dfrac{16}{x};\ [1, 3]$

21. $f(x) = 3 - 2x;\ [-1, 1]$

22. $f(x) = x^2 - 4x + 3;\ [0, 2]$

23. $f(x) = 5 - 12x - 9x^2;\ [-1, 1]$

24. $f(x) = 2x^2 - 4x + 7;\ [0, 2]$

25. $f(x) = x^3 - 3x^2 - 9x + 5;\ [-2, 4]$

26. $f(x) = x^3 + x;\ [-1, 2]$

27. $f(x) = 3x^5 - 5x^3;\ [-2, 2]$

28. $f(x) = |2x - 3|;\ [1, 2]$

29. $f(x) = 5 + |7 - 3x|;\ [1, 5]$

30. $f(x) = |x + 1| + |x - 1|;\ [-2, 2]$

31. $f(x) = 50x^3 - 105x^2 + 72x;\ [0, 1]$

32. $f(x) = 2x + \dfrac{1}{2x};\ [1, 4]$

33. $f(x) = \dfrac{x}{x + 1};\ [0, 3]$

34. $f(x) = \dfrac{x}{x^2 + 1};\ [0, 3]$

35. $f(x) = \dfrac{1 - x}{x^2 + 3};\ [-2, 5]$

36. $f(x) = 2 - \sqrt[3]{x};\ [-1, 8]$

37. $f(x) = x\sqrt{1 - x^2};\ [-1, 1]$

38. $f(x) = x\sqrt{4 - x^2};\ [0, 2]$

39. $f(x) = x(2 - x)^{1/3};\ [1, 3]$

40. $f(x) = x^{1/2} - x^{3/2};\ [0, 4]$

41. Suppose that $f(x) = Ax + B$ is a linear function and that $A \neq 0$. Explain why the maximum and minimum values of f on a closed interval $[a, b]$ must occur at the endpoints of the interval.

42. Suppose that f is continuous on $[a, b]$ and differentiable on (a, b) and that $f'(x)$ is never zero at any point of (a, b). Explain why the maximum and minimum values of f must occur at the endpoints of the interval $[a, b]$.

43. Explain why every real number is a critical point of the greatest integer function $f(x) = [\![x]\!]$.

44. Prove that every quadratic function

$$f(x) = ax^2 + bx + c \quad (a \neq 0)$$

has exactly one critical point on the real line.

45. Explain why the cubic polynomial function

$$f(x) = ax^3 + bx^2 + cx + d \quad (a \neq 0)$$

can have either two, one, or no critical points on the real line. Produce examples that illustrate each of the three cases.

46. Define $f(x)$ to be the distance from x to the nearest integer. What are the critical points of f?

In Problems 47 through 52, match the given graph of the function with the graph of its derivative f' from those in Fig. 3.5.12, parts (a) through (f).

We square both sides, clear the equation of fractions, and simplify. The result is

$$\begin{aligned} x^2[3600 + (200 - x)^2] &= (200 - x)^2(8100 + x^2); \\ 3600x^2 &= 8100(200 - x)^2; \quad \text{(Why?)} \\ 60x &= 90(200 - x); \\ 150x &= 18000; \\ x &= 120. \end{aligned}$$

Thus the cow should proceed directly to the point P located 120 ft along the water trough. ■

These examples indicate that the closed-interval maximum-minimum method is applicable to a wide range of problems. Indeed, applied optimization problems that seem as different as light rays and cows may have essentially identical mathematical models. This is only one illustration of the power of generality that calculus exploits so effectively.

3.6 PROBLEMS

1. Find two positive real numbers x and y such that their sum is 50 and their product is as large as possible.
2. Find the maximum possible area of a rectangle of perimeter 200 m.
3. A rectangle with sides parallel to the coordinate axes has one vertex at the origin, one on the positive x-axis, one on the positive y-axis, and its fourth vertex in the first quadrant on the line with equation $2x + y = 100$ (Fig. 3.6.15). What is the maximum possible area of such a rectangle?
4. A farmer has 600 m of fencing with which to enclose a rectangular pen adjacent to a long existing wall. He will use the wall for one side of the pen and the available fencing for the remaining three sides. What is the maximum area that can be enclosed in this way?
5. A rectangular box has a square base with edges at least 1 in. long. It has no top, and the total area of its five sides is 300 in.^{2} (Fig. 3.6.16). What is the maximum possible volume of such a box?
6. If x is in the interval $[0, 1]$, then $x - x^2$ is not negative. What is the maximum value that $x - x^2$ can have on that interval? In other words, what is the greatest amount by which a real number can exceed its square?
7. The sum of two positive numbers is 48. What is the smallest possible value of the sum of their squares?
8. A rectangle of fixed perimeter 36 is rotated around one of its sides, thus sweeping out a figure in the shape of a right circular cylinder (Fig. 3.6.17). What is the maximum possible volume of that cylinder?

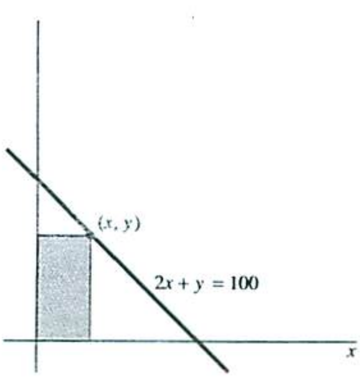

Fig. 3.6.15 The rectangle of Problem 3

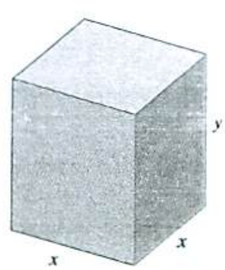

Fig. 3.6.16 A box with square base and volume $V = x^2y$ (Problems 5, 17, and 20)

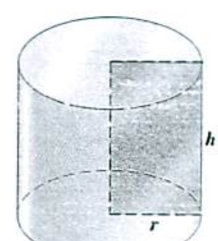

Fig. 3.6.17 The rectangle and cylinder of Problem 8

9. The sum of two nonnegative real numbers is 10. Find the minimum possible value of the sum of their cubes.
10. Suppose that the strength of a rectangular beam is proportional to the product of the width and the square of the height of its cross section. What shape beam should be cut from a cylindrical log of radius r to achieve the greatest possible strength?
11. A farmer has 600 yd of fencing with which to build a rectangular corral. Some of the fencing will be used to con-

struct two internal divider fences, both parallel to the same two sides of the corral (Fig. 3.6.18). What is the maximum possible total area of such a corral?

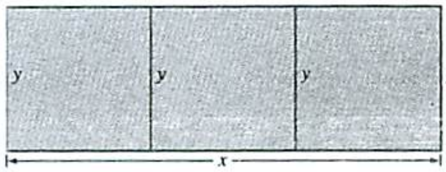

Fig. 3.6.18 The divided corral of Problem 11

12. Find the maximum possible volume of a right circular cylinder if its total surface area—including both circular ends—is 150π.

13. Find the maximum possible area of a rectangle with diagonals of length 16.

14. A rectangle has a line of fixed length L reaching from one vertex to the midpoint of one of the far sides (Fig. 3.6.19). What is the maximum possible area of such a rectangle?

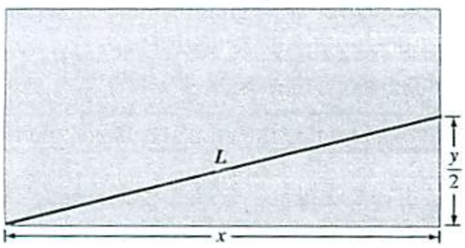

Fig. 3.6.19 The rectangle of Problem 14

15. The volume V (in cubic centimeters) of 1 kg of water at temperature T between 0°C and 30°C is very closely approximated by

$$V = 999.87 - (0.06426)T + (0.0085043)T^2 - (0.0000679)T^3.$$

At what temperature does water have its maximum density?

16. What is the maximum possible area of a rectangle with a base that lies on the x-axis and with two upper vertices that lie on the graph of the equation $y = 4 - x^2$ (Fig. 3.6.20)?

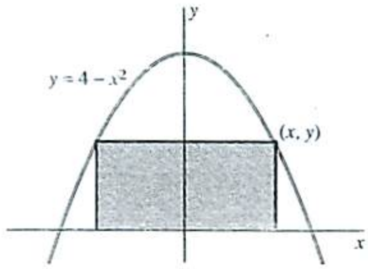

Fig. 3.6.20 The rectangle of Problem 16

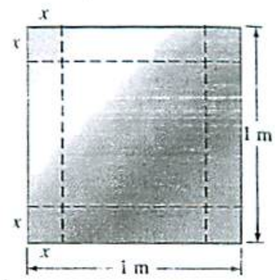

Fig. 3.6.21 One of the three 1-m squares of Problem 19

17. A rectangular box has a square base with edges at least 1 cm long. Its total surface area is 600 cm^2. What is the largest possible volume that such a box can have?

18. You must make a cylindrical can with a bottom but no top from 300π in.^{2} of sheet metal. No sheet metal will be wasted; you are allowed to order a circular piece of any size for its base and any appropriate rectangular piece to make into its curved side as long as the given conditions are met. What is the greatest possible volume of such a can?

19. Three large squares of tin, each with edges 1 m long, have four small, equal squares cut from their corners. All twelve resulting small squares are to be the same size (Fig. 3.6.21). The three large cross-shaped pieces are then folded and welded to make boxes with no tops, and the twelve small squares are used to make two small cubes. How should this be done to maximize the total volume of all five boxes?

20. Suppose that you are to make a rectangular box with a square base from two different materials. The material for the top and four sides of the box costs \$1/ft^2; the material for the base costs \$2/ft^2. Find the dimensions of the box of greatest possible volume if you are allowed to spend \$144 for the material to make it.

21. A piece of wire 80 in. long is cut into at most two pieces. Each piece is bent into the shape of a square. How should this be done to minimize the sum of the area(s) of the square(s)? to maximize it?

22. A wire of length 100 cm is cut into two pieces. One piece is bent into a circle, the other into a square. Where should the cut be made to maximize the sum of the areas of the square and the circle? to minimize that sum?

23. A farmer has 600 m of fencing with which she plans to enclose a rectangular pasture adjacent to a long existing wall. She plans to build one fence parallel to the wall, two to form the ends of the enclosure, and a fourth (parallel to the ends of the enclosure) to divide it equally. What is the maximum area that can be enclosed?

24. A zookeeper needs to add a rectangular outdoor pen to an animal house with a corner notch, as shown in Fig. 3.6.22. If

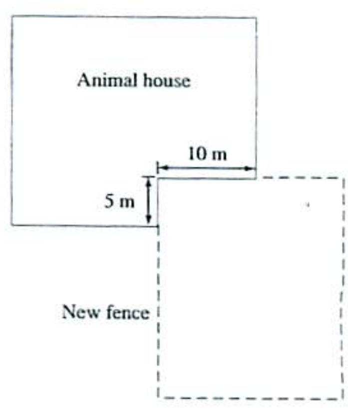

Fig. 3.6.22 The rectangular pen of Problem 24

points in Fig. 4.7.11. We could zoom in closer to each of these solutions, or we could use a calculator or computer **SOLVE** command to get the approximations -5.4303, 0.3152, 0.6503, and 1.3937. The larger view shown in Fig. 4.7.17 convinces us that we've found *all* the inflection points of $y = f(x)$. In particular, we see that $y = f(x)$ is concave upward to the right of the inflection point $x \approx -5.4303$, where the denominator in Eq. (10) is negative (why?), and is concave downward just to its left (consistent with what we see in Fig. 4.7.15).

This thorough analysis of the graph of the function f of Eq. (8) involves a certain amount of manual labor—just to calculate and simplify the derivatives in Eqs. (9) and (10) unless we use a computer algebra system for this task—but would be very challenging without the use of a graphing calculator or computer.

4.7 PROBLEMS

Investigate the limits in Problems 1 through 16.

1. $\lim_{x\to+\infty} \dfrac{x}{x+1}$

2. $\lim_{x\to-\infty} \dfrac{x^2+1}{x^2-1}$

3. $\lim_{x\to 1} \dfrac{x^2+x-2}{x-1}$

4. $\lim_{x\to 1} \dfrac{x^2-x-2}{x-1}$

5. $\lim_{x\to+\infty} \dfrac{2x^2-1}{x^2-3x}$

6. $\lim_{x\to-\infty} \dfrac{x^2+3x}{x^3-5}$

7. $\lim_{x\to-1} \dfrac{x^2+2x+1}{(x+1)^2}$

8. $\lim_{x\to+\infty} \dfrac{5x^3-2x+1}{7x^3+4x^2-2}$

9. $\lim_{x\to 4} \dfrac{x-4}{\sqrt{x}-2}$

10. $\lim_{x\to+\infty} \dfrac{2x+1}{x-x\sqrt{x}}$

11. $\lim_{x\to-\infty} \dfrac{8-\sqrt[3]{x}}{2+x}$

12. $\lim_{x\to+\infty} \dfrac{2x^2-17}{x^3-2x+27}$

13. $\lim_{x\to+\infty} \sqrt{\dfrac{4x^2-x}{x^2+9}}$

14. $\lim_{x\to-\infty} \dfrac{\sqrt[3]{x^3-8x+1}}{3x-4}$

15. $\lim_{x\to-\infty} \left(\sqrt{x^2+2x}-x\right)$

16. $\lim_{x\to-\infty} \left(2x-\sqrt{4x^2-5x}\right)$

Apply your knowledge of limits and asymptotes to match each function in Problems 17 through 28 with its graph-with-asymptotes in one of the twelve parts of Fig. 4.7.18.

17. $f(x) = \dfrac{1}{x-1}$

18. $f(x) = \dfrac{1}{1-x}$

19. $f(x) = \dfrac{1}{(x-1)^2}$

20. $f(x) = -\dfrac{1}{(1-x)^2}$

21. $f(x) = \dfrac{1}{x^2-1}$

22. $f(x) = \dfrac{1}{1-x^2}$

23. $f(x) = \dfrac{x}{x^2-1}$

24. $f(x) = \dfrac{x}{1-x^2}$

25. $f(x) = \dfrac{x}{x-1}$

26. $f(x) = \dfrac{x^2}{x^2-1}$

27. $f(x) = \dfrac{x^2}{x-1}$

28. $f(x) = \dfrac{x^3}{x^2-1}$

Sketch by hand the graph of each function in Problems 29 through 54. Identify and label all extrema, inflection points, intercepts, and asymptotes. Show the concave structure clearly as well as the behavior of the graph for $|x|$ large and for x near any discontinuities of the function.

29. $f(x) = \dfrac{2}{x-3}$

30. $f(x) = \dfrac{4}{5-x}$

31. $f(x) = \dfrac{3}{(x+2)^2}$

32. $f(x) = -\dfrac{4}{(3-x)^2}$

33. $f(x) = \dfrac{1}{(2x-3)^3}$

34. $f(x) = \dfrac{x+1}{x-1}$

35. $f(x) = \dfrac{x^2}{x^2+1}$

36. $f(x) = \dfrac{2x}{x^2+1}$

37. $f(x) = \dfrac{1}{x^2-9}$

38. $f(x) = \dfrac{x}{4-x^2}$

39. $f(x) = \dfrac{1}{x^2+x-6}$

40. $f(x) = \dfrac{2x^2+1}{x^2-2x}$

41. $f(x) = x + \dfrac{1}{x}$

42. $f(x) = 2x + \dfrac{1}{x^2}$

43. $f(x) = \dfrac{x^2}{x-1}$

44. $f(x) = \dfrac{2x^3-5x^2+4x}{x^2-2x+1}$

45. $f(x) = \dfrac{1}{(x-1)^2}$

46. $f(x) = \dfrac{1}{x^2-4}$

47. $f(x) = \dfrac{x}{x+1}$

48. $f(x) = \dfrac{1}{(x+1)^3}$

49. $f(x) = \dfrac{1}{x^2-x-2}$

50. $f(x) = \dfrac{1}{(x-1)(x+1)^2}$

51. $f(x) = \dfrac{x^2-4}{x}$

52. $f(x) = \dfrac{x}{x^2-1}$

53. $f(x) = \dfrac{x^3-4}{x^2}$

54. $f(x) = \dfrac{x^2+1}{x-2}$

In Problems 55 through 60, you can determine by inspection the x-intercepts as well as the vertical and horizontal asymptotes of the curve $y = f(x)$. First sketch the graph by hand, using this information, and without calculating any derivatives. Then use a calculator or computer to locate accurately the critical and inflection points of $f(x)$. Finally, use a calculator or computer to produce graphs that display the major features of the curve.

54. $\lim_{x \to 0^+} x \ln x$ (*Suggestion:* Substitute $x = 1/u$.)

55. $\lim_{x \to 0^+} \sqrt{x} \ln x$

56. $\lim_{x \to \infty} \dfrac{(\ln x)^2}{x}$

57. Use the method of Example 4 to deduce that

$$\int \cot x \, dx = \ln|\sin x| + C.$$

58. Find a formula for $f^{(n)}(x)$, given $f(x) = \ln x$ (n denotes a positive integer).

59. The heart rate R (in beats per minute) and weight W (in pounds) of various mammals were measured, with the results shown in Fig. 7.2.13. Use the method of Example 9 to find a relation between the two of the form $R = kW^m$.

W	25	67	127	175	240	975
R	131	103	88	81	75	53

Fig. 7.2.13 Data for Problem 59

60. During the adiabatic expansion of a certain diatomic gas, its volume V (in liters) and pressure p (in atmospheres) were measured, with the results shown in Fig. 7.2.14. Use the method of Example 9 to find a relation between V and p of the form $p = kV^m$.

V	1.46	2.50	3.51	5.73	7.26
p	28.3	13.3	8.3	4.2	3.0

Fig. 7.2.14 Data for Problem 60

In Problems 61 and 62, graph (on a single calculator or computer screen) the functions on either side of the given equation to locate its single positive solution. Then determine numerically the value of this solution accurate to three decimal places.

61. $\ln x = 5 - x$

62. $\ln x = \dfrac{1}{x}$

In Problems 63 through 66, graph (on a single calculator or computer screen) the functions on both sides of the given equation to determine how many positive solutions the equation has. Then determine numerically the value of each solution accurate to three decimal places.

63. $2 \ln x = x - 2$

64. $4 \ln x = (x - 3)^2$

65. $2 \ln x = 3 \sin 2x$

66. $2 \ln x = 5 \cos x$

67. Find graphically the coordinates (accurate to three decimal places) of the intersection point of the graphs $y = \ln x$ and $y = x^{1/10}$ shown in Fig. 7.2.7.

68. Determine a viewing rectangle that reveals the second intersection point (with $x > 10$) of the graphs $y = \ln x$ and $y = x^{1/10}$. Then determine graphically the first three digits of the larger solution x of the equation $\ln x = x^{1/10}$ (thus writing this solution in the form $p.qr \times 10^k$).

69. Approximate numerically the area of the region that lies beneath the curve $y = 5 \ln x - 2x + 3$ and above the x-axis. You will first need to estimate graphically the x-intercepts of this curve, then integrate numerically (perhaps using the numerical integration facility of your calculator or computer).

70. Approximate numerically the area of the region bounded by the curves $y = 10 \ln x$ and $y = (x - 5)^2$. You will first need to estimate graphically the x-coordinates of the intersection points of the two curves, then integrate numerically (perhaps using the numerical integration facility of your calculator or computer).

71. Substitute $y = x^p$ and then apply Eq. (13) to show that

$$\lim_{x \to \infty} \frac{\ln x}{x^p} = 0 \quad \text{if } 0 < p < 1.$$

72. Deduce from the result of Problem 71 that

$$\lim_{x \to \infty} \frac{(\ln x)^k}{x} = 0 \quad \text{if } k > 0.$$

73. Substitute $y = 1/x$ and then apply Eq. (13) to show that

$$\lim_{x \to 0^+} x^k \ln x = 0 \quad \text{if } k > 0.$$

Use the limits in Problems 71 through 73 to help you sketch the graphs, for $x > 0$, of the functions given in Problems 74 through 77.

74. $y = x \ln x$

75. $y = x^2 \ln x$

76. $y = \sqrt{x} \ln x$

77. $y = \dfrac{\ln x}{\sqrt{x}}$

78. Problem 26 of Section 5.9 asks you to show by numerical integration that

$$\int_1^{2.7} \frac{dx}{x} < 1 < \int_1^{2.8} \frac{dx}{x}.$$

Explain carefully why this result proves that $2.7 < e < 2.7$.

79. If n moles of an ideal gas expand at *constant* temperature T, then the pressure and volume satisfy the ideal-gas equation $pV = nRT$ (n and R are constants). With the aid of Problem 22 in Section 6.6, show that the work W done by the gas in expanding from volume V_1 to volume V_2 is

$$W = nRT \ln \frac{V_2}{V_1}.$$

80. "Gabriel's horn" is obtained by revolving around the x-axis the curve $y = 1/x$, $x \geqq 1$ (Fig. 7.2.15). Let A_b denote its surface area from $x = 1$ to $x = b$. Show that $A_b \geqq 2\pi \ln b$, so—as a consequence—$A_b \to +\infty$ as

7.3 PROBLEMS

Differentiate the functions in Problems 1 through 30.

1. $f(x) = e^{2x}$
2. $f(x) = e^{3x-1}$
3. $f(x) = e^{x^2} = \exp(x^2)$
4. $f(x) = e^{4-x^3}$
5. $f(x) = e^{1/x^2}$
6. $f(x) = x^2 e^{x^3}$
7. $g(t) = te^{\sqrt{t}}$
8. $g(t) = (e^{2t} + e^{3t})^7$
9. $g(t) = (t^2 - 1)e^{-t}$
10. $g(t) = \sqrt{e^t - e^{-t}}$
11. $g(t) = e^{\cos t}$
12. $f(x) = xe^{\sin x}$
13. $f(x) = \cos(1 - e^{-x})$
14. $f(x) = \sin^2(e^{-x})$
15. $f(x) = \ln(x + e^{-x})$
16. $f(x) = e^x \cos 2x$
17. $f(x) = e^{-2x}\sin 3x$
18. $g(t) = \ln(te^{t^2})$
19. $g(t) = 3(e^t - \ln t)^5$
20. $g(t) = \sin(e^t)\cos(e^{-t})$
21. $f(x) = \dfrac{2 + 3x}{e^{4x}}$
22. $g(t) = \dfrac{1 + e^t}{1 - e^t}$
23. $g(t) = \dfrac{1 - e^{-t}}{t}$
24. $f(x) = e^{-1/x}$
25. $f(x) = \dfrac{1 - x}{e^x}$
26. $f(x) = e^{\sqrt{x}} + e^{-\sqrt{x}}$
27. $f(x) = e^{e^x}$
28. $f(x) = \sqrt{e^{2x} + e^{-2x}}$
29. $f(x) = \sin(2e^x)$
30. $f(x) = \cos(e^x + e^{-x})$

In Problems 31 through 35, find dy/dx by implicit differentiation.

31. $xe^y = y$
32. $\sin(e^{xy}) = x$
33. $e^x + e^y = e^{xy}$
34. $x = ye^y$
35. $e^{x-y} = xy$

Find the antiderivatives indicated in Problems 36 through 53.

36. $\int e^{3x}\,dx$
37. $\int e^{1-2x}\,dx$
38. $\int xe^{x^2}\,dx$
39. $\int x^2 e^{3x^3-1}\,dx$
40. $\int \sqrt{x}\,e^{2x\sqrt{x}}\,dx$
41. $\int \dfrac{e^{2x}}{1 + e^{2x}}\,dx$
42. $\int (\cos x)e^{\sin x}\,dx$
43. $\int (\sin 2x)e^{1-\cos 2x}\,dx$
44. $\int (e^x + e^{-x})^2\,dx$
45. $\int \dfrac{x + e^{2x}}{x^2 + e^{2x}}\,dx$
46. $\int e^{2x+3}\,dx$
47. $\int te^{-t^2/2}\,dt$
48. $\int x^2 e^{1-x^3}\,dx$
49. $\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$
50. $\int \dfrac{e^{1/t}}{t^2}\,dt$
51. $\int \dfrac{e^x}{1 + e^x}\,dx$
52. $\int \exp(x + e^x)\,dx$
53. $\int \sqrt{x}\exp\left(-\sqrt{x^3}\right)dx$

Apply Eq. (18) to evaluate (in terms of the exponential function) the limits in Problems 54 through 58.

54. $\lim_{n\to\infty}\left(1 - \dfrac{1}{n}\right)^n$
55. $\lim_{n\to\infty}\left(1 + \dfrac{2}{n}\right)^n$
56. $\lim_{n\to\infty}\left(1 + \dfrac{2}{3n}\right)^n$
57. $\lim_{h\to 0}(1 + h)^{1/h}$
58. $\lim_{h\to 0}(1 + 2h)^{1/h}$ (*Suggestion*: Substitute $k = 2h$.)

Evaluate the limits in Problems 59 through 62 by applying the fact that

$$\lim_{x\to\infty} x^k e^{-x} = 0.$$

59. $\lim_{x\to\infty}\dfrac{e^x}{x}$
60. $\lim_{x\to\infty}\dfrac{e^x}{\sqrt{x}}$
61. $\lim_{x\to\infty}\dfrac{e^{\sqrt{x}}}{x}$
62. $\lim_{x\to\infty} x^2 e^{-x}$

In Problems 63 through 65, sketch the graph of the given equation. Show and label all extrema, inflection points, and asymptotes; show the concave structure clearly.

63. $y = x^2 e^{-x}$
64. $y = x^3 e^{-x}$
65. $y = \exp(-x^2)$
66. Find the area under the graph of $y = e^x$ from $x = 0$ to $x = 1$.
67. Find the volume generated by revolving the region of Problem 66 around the x-axis.
68. Let R be the plane figure bounded below by the x-axis, above by the graph of $y = \exp(-x^2)$, and on the sides by the vertical lines at $x = 0$ and $x = 1$ (Fig. 7.3.7). Find the volume generated by rotating R around the y-axis.

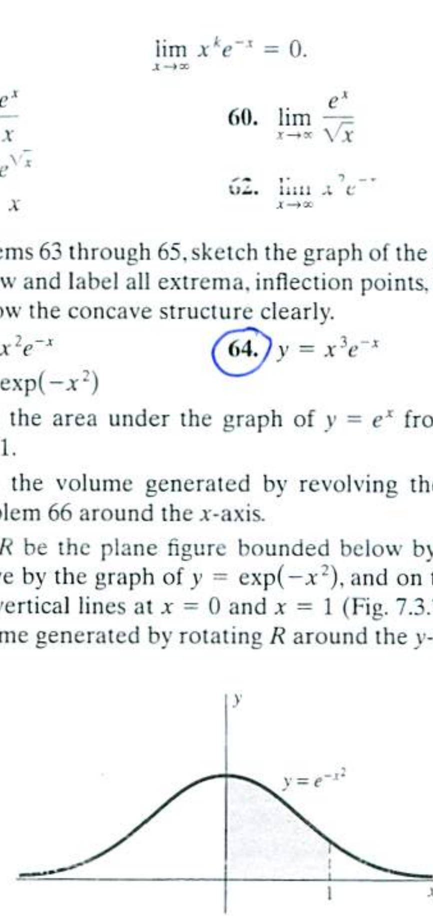

Fig. 7.3.7 The region of Problem 68

69. Find the length of the curve $y = \frac{1}{2}(e^x + e^{-x})$ from $x = 0$ to $x = 1$.
70. Find the area of the surface generated by revolving around the x-axis the curve of Problem 69 (Fig. 7.3.8).

In Problems 71 and 72, graph (on a single calculator or computer screen) the functions on both sides of the given equation to locate its single positive solution. Then determine numerically the value of this solution accurate to three decimal places.

71. $e^{-x} = x - 1$
72. $e^{-x} = \ln x$

In Problems 73 through 76, graph (on a single calculator or computer screen) the functions on both sides of the given equation to determine how many positive solutions it has. Then determine numerically the value of each solution accurate to three decimal places.