Engineering and Science Mathematics 1A

Midterm Exam I

October 5, 2010

1. Compute the following limits provided they exist.

   (a) $\lim_{a\to 1} \dfrac{|a-1|}{a-1}$

   (b) $\lim_{a\to 1} \dfrac{a^3-3a^2+3a-1}{a^2-2a+1}$

   (c) $\lim_{a\to 0} \sin a \cos\dfrac{1}{a}$

   (5+5+5)

2. The function

   $$\phi(x) = x \ln|x|$$

   is not defined at $x = 0$.

   (a) Can you assign a value to $\phi(0)$ in such a way to make ϕ continuous?

   (b) Can you assign a value to $\phi'(0)$ in such a way to make ϕ' continuous?

   (5+5)

3. Compute the derivative of $g(x) = \sqrt{x}$ by

   (a) using the definition of the derivative as the limit of a difference quotient,

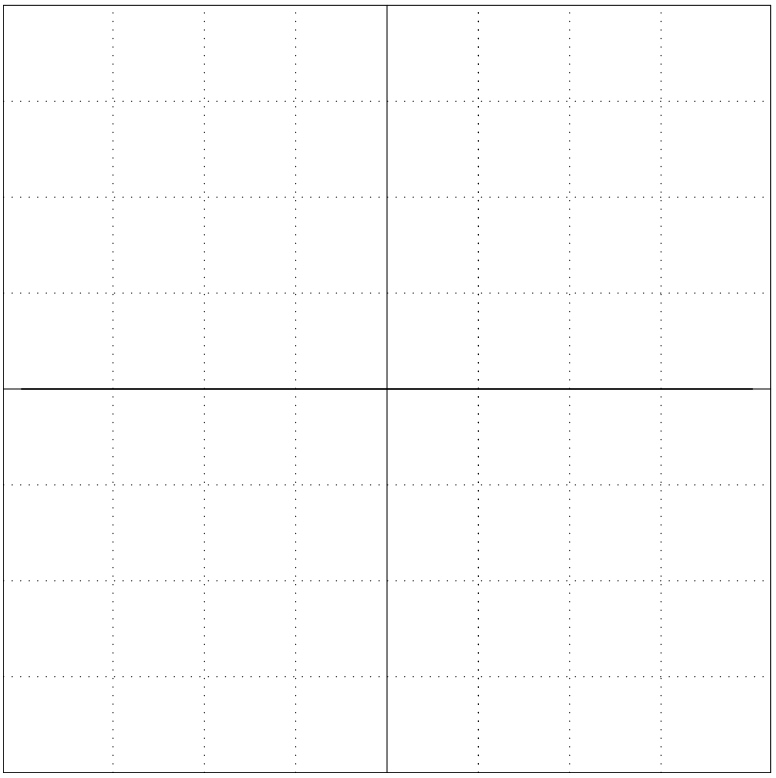
   (b) using the power rule of differentiation.

   Confirm that the two results coincide. (5+10)

4. Consider the function

   $$f(x) = x e^{-x}.$$

   Find horizontal and vertical asymptotes, local minima, local maxima, and inflection points of f. Identify the regions where the graph of f is concave upward or concave downward. Finally, sketch the graph into the coordinate system provided. (10)



5. A commuter train carries 400 passengers each day at a ticket price of €1.00 per person. Market research reveals that for each 5 cent increase/decrease of the price 50 fewer/more people would take the train. What fare should be charged to maximize revenue? (10)