Engineering and Science Mathematics 1A

Midterm Exam II

November 11, 2010

Some trigonometric identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin 2x = 2\sin x\cos x$$
$$1 + \tan^2\theta = \sec^2\theta \qquad \cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$1 + \cot^2\theta = \csc^2\theta \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

Useful integrals:

$$\int \frac{du}{\sqrt{1-u^2}} = \arcsin u + C$$
$$\int \frac{du}{1+u^2} = \arctan u + C = -\operatorname{arccot} u + C'$$
$$\int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec}|u| + C$$
$$\int \sec u\,du = \ln\left|\sec u + \tan u\right| + C$$

1. Find all points (x, y) on the graph of $x^2 - xy + y^2 = 1$ where the tangent line is horizontal. (10)

2. Compute the following indefinite integrals

   (a) $\displaystyle\int \sin x \sec x\,dx$

   (b) $\displaystyle\int x(\ln x)^3\,dx$

   (c) $\displaystyle\int \frac{\cos x}{\sqrt{4-\sin^2 x}}\,dx$

(d) $\int x^3 e^{-x^2}\,dx$

(e) $\int \dfrac{4x^2+x+1}{4x^3+x}\,dx$

(10+10+10+10+10)

3. Compute the average value of the function $f(x) = \sin 2x$ on the interval $[0, \pi]$. (5)

4. Does the improper integral

$$\int_1^\infty \frac{1}{x}\,dx$$

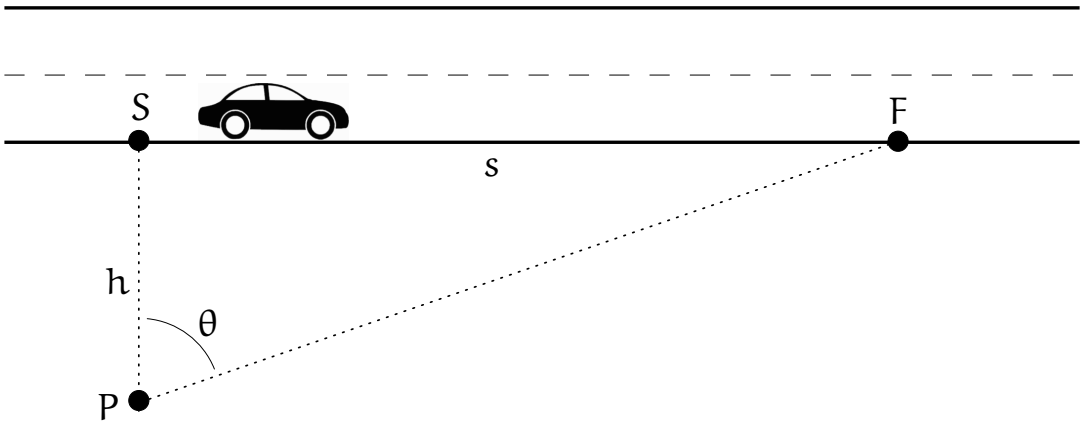
converge? If so, compute its value. (10)

5. Compute the area between the x-axis and the graph of

$$f(x) = \frac{1}{9+x^2}\,.$$

(10)

6. A policeman is positioned at point P alongside a straight stretch of road to catch speeding cars. He employs the following method: When a car passes the point S closest to him, he starts his stopwatch. He then looks through a spotting scope, mounted at an angle θ toward the road. When the car comes into view at point F, he stops the clock. The setting is shown in this figure:



Suppose that the distance of the policeman to the road is h, that the angular resolution of the spotting scope is $\Delta\theta$ (in other words, $\Delta\theta$ is the expected measurement error of the angle θ), and that all other measurement errors are negligible.

(a) Use linear approximation to derive a formula for the expected error Δs in the computed traveled distance s.

(b) Then derive a formula for the expected error Δv in the computed velocity v of the car.

(c) What angle θ should the policeman use to minimize the error in the velocity?

Hint: In part (c) you should find that the optimal angle is independent of the concrete numerical values of h, v, and $\Delta\theta$.

(5+5+5)