Derivatives Lab

Session 10

October 24, 2011

1. Recall that for a binomial tree with probability for upward movement p, the expected single period return is
$$\hat{\mu} = p \ln \frac{u}{d} + \ln d$$
with single period variance
$$\hat{\sigma}^2 = p\,(1-p)\,\ln^2 \frac{u}{d}\,.$$
Recall further that, to obtain a limit as $n \to \infty$, we must have
$$\hat{\mu} \sim \mu\,\frac{T}{n}$$
and
$$\hat{\sigma}^2 \sim \sigma^2\,\frac{T}{n}$$
where T is the total duration in years, μ the annual continuously compounded rate of return, and σ the so-called annualized volatility.

   So far, we have satisfied these relationships choosing
$$u = \mathrm{e}^{\sigma\sqrt{T/n}}\,,$$
$$d = \mathrm{e}^{-\sigma\sqrt{T/n}}\,,$$
$$p = \frac{1}{2} + \frac{1}{2}\,\frac{\mu}{\sigma}\,\sqrt{\frac{T}{n}}\,.$$

   What expressions do you obtain by fixing $p = 1/2$ (rather than $u = 1/d$)? Does the binomial tree model compute the same option prices?

   (a) Modify your binomial tree program to try this out.

   (b) Why does it work?

2. Compare your call option prices against those computed by the Black–Scholes formula. Does the error scale like a power of n? (Plot the logarithm of the error vs. n. Do you obtain a straight line?)