# Derivatives Lab

Session 6

September 20, 2011

1. Show that, under continuous compounding, the implied forward rate equals

$$S(i,j) = \frac{j\,S(j) - i\,S(i)}{j-i}\,,$$

so that the implied single-period forward rate reads

$$S(i,i+1) = (i+1)\,S(i+1) - i\,S(i)\,.$$

*Hint:* The discount factor over $n$ periods is $\exp(-n\,S(n))$.

Extend your program from Session 5, Exercise 2 to also plot the implied one-period forward rate into the same coordinate system as computed by the above formula, and the "instantaneous forward rate" given in the ECB table.

2. Show how you can construct a portfolio which is equivalent to the issue of a zero coupon bond at period $i$ with maturity $j$ at today's implied forward rate $S(i,j)$. (Such a portfolio is called a *forward contract.*)

3. Suppose that level coupon bonds of all maturities, coupon rates, and par values are freely tradeable at their fair market price. Suppose you wish to immunize a future liability at the end of period $n$. Even if interest rates never change, a portfolio which is immunized at the beginning of the first period, the Macaulay duration will drift, so the portfolio will need to be rebalanced. You may proceed as follows.

   - Take a bond with maturity $2n-2$ (or maturity 1 if $n=1$).[1]
   - Ensure that MD $= n$ by adjusting the coupon rate $c$.
   - Match the liability by adjusting the par value $F$.
   - At the end of each period, sell the bond at its fair market price and buy a new bond, immunized according to the above procedure.

[1] The initial instructions suggested taking maturity $2n$. However, in this case, it is possible that the coupon rate becomes negative. Can you prove that this does not happen for any maturity between $n$ and $2n-2$?

- Do you gain or lose money relative to meeting the liability by buying a zero coupon bond?
- Repeat the analysis with random changes of the interest rate within each period.

4. Look on the internet for yield rates of Greek treasury bonds. Plot a curve showing the assumed default probability vs. years from now under the analysis that in the event of default, no money is repaid.