

Perspectives of Mathematics I

Homework 3

due November 16, 2011

1. Find the pure-strategy Nash-equilibria for the two-player game with payoff matrix

		Player 2		
		l	c	r
Player 1	U	1, 1	2, 0	3, 0
	M	0, 2	3, 3	0, 0
	D	0, 3	0, 0	4, 4

2. Consider the two-player game with payoff matrix

		Player 2	
		l	r
Player 1	U	3, 1	0, 0
	D	0, 0	1, 3

- (a) Are there pure-strategy Nash equilibria?
(b) What are the mixed-strategy Nash equilibria?
(c) Sketch the best response functions for case (b).
3. Consider the following slight modification of the Cournot game considered in class. Two firms produce quantities q_1 and q_2 of the same good with respective unit cost per unit c_1 and c_2 with no fixed costs. The market price is assumed to satisfy the simple inverse linear relation $P = a - q_1 - q_2$. Now, Firm 1's objective is to maximize its market share without making a loss. Firm 2's objective is to maximize profit. Compute both firms' best response functions and describe the Nash equilibria.
4. (a) Show that the relation

$$f(x) = \begin{cases} (\frac{1}{2}, 1] & \text{if } x < \frac{1}{2} \\ [0, \frac{1}{2}) & \text{if } x \geq \frac{1}{2} \end{cases}$$

on the interval $[0, 1]$ does not have a fixed point $x \in f(x)$.

- (b) Which condition of Kakutani's theorem is violated?