Derivatives Lab

Session 12

October 29, 2012

1. Compute an ensemble of standard Brownian paths $W(t)$ over the interval $[0, 1]$ partitioned into $N = 500$ time steps. Plot the empirically determied mean and standard deviation of the ensemble as a function of time.

2. Similarly, compute an ensemble of geometric Brownian paths

$$S(t) = \exp((\mu - \tfrac{1}{2}\,\sigma^2)\,t + \sigma\,W(t))$$

   with $\mu = 0.05$ and $\sigma = 0.3$ and plot mean and standard deviation as a function of time on the interval $[0, 1]$.

3. Compute the corresponding stock price paths which underlie the binomial tree model using the parameters from Problem 2 and compare their mean and standard deviation with those obtained from geometric Brownian motion.

4. Use the paths so obtained in a Monte–Carlo valuation of a European call option with $K = 0.9$, time of maturity $T = 1.0$ and risk free rate $r = \mu$. Compare your result against the Black–Scholes price by plotting the deviation from the Black–Scholes price against the number of samples in a doubly logarithmic plot.

   What is the order of the Monte–Carlo method as a function of the number of samples?